# Grammar on mathematical principles 

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This paper ${ }^{1}$ presents a grammatical theory which has certain mathematical properties, and which produces the sentences of a language by means of two simple processes, word-entry and entry-reduction, with the meaning of each sentence being indicated directly by its construction. The reason for seeking a mathematical control on grammatical analysis lies partly in the inherent possibilities of such a connexion, and partly in the complex and somewhat haphazard nature of grammar, which makes one wonder if there is not something simpler and more principled involved. In the event, the system of grammatical analysis which is reached here is not an alternative to descriptions of grammatical patterns, but rather a complement to them; for in producing the sentences it locates and explicates those patterns as resultants of the two processes, but without cataloguing or arranging them.

The essential problem was to find one or more relations (or operations) on the set of sentences in respect to which that set - or, as it turned out, an extension of that set - is closed. To meet this problem we will establish certain overt relations among English sentences, which can then be shown to hold also in less noticeable situations. The result will be the set of grammatically possible sentences; and it is this set which will be seen to have the desired mathematical properties. Data for the analysis presented here are to be found in Harris (1976) and to appear; further discussion is in Harris (1976a, b).

## i. Reduction

First, we note certain relations among subsets of sentences, where one can say that a word has been reduced in its phonemic shape. A simple case is that of pronouns: we can save pronouns from being something unique in grammar, if we describe pronominalization not as replacement of a noun by a pronoun, but as reduction of a repeated noun to a short phonemic sequence - that which is called 'pronoun'. The reason pronouns are thought of as separate words is that there is a systemic degeneracy: different words, when repeated, are reduced to the same phonemic sequence. Similarly, repeated words can be reduced to zero ('zeroed'), as in Bach wrote cantatas and Mozart operas from Bach wrote cantatas and Mozart wrote operas, and in I asked to be last from I asked that I be last. Here too different words are reduced to the same zero. 'Pronoun' and 'zero' may therefore be used as verbs, like 'reduce'.

[^0]In certain sentence-positions, indefinite nouns such as things, people, someone are zeroable. We can take We opposed building the monument, from We opposed people's building the monument, and He read for his examinations from He read things (or: the relevant things) for his examinations. Assuming here the existence and then zeroing of things is preferable to saying that read is intransitive as well as transitive; for read is intransitive only in contexts in which an object of it can be said to have been zeroed.

There are also cases where it is clear that a particular word (or one of its synonyms) is the indicated word in the given environment, and has been zeroed. Thus the object of expect is normally a sentence: I expect that Yohn will be late, $I$ expect fohn to leave, or to come, etc. When we see I expect fohn, then rather than say that expect can also have noun as object, we say that to be here, or to come, has been zeroed in the sentential object of expect. This, not only because the meaning of I expect fohn is thereby obtained, but also because nouns can be objects of expect precisely to the extent that they can be subjects of be here, come: We no more say *I expect time, *I expect the universe, than we say *Time is here, *The universe comes.

Also, when we compare my friend, the ambassador, with my friend, who is the ambassador, and this book here with this book which is here, and a correct copy with a copy which is correct, we can say that which is, who is is zeroed, with required shifting of adjectival words to before the noun.

A more complicated case of zeroing of particular words is seen in the compound stress. All compound words, $N-X$, aside from a few petrified cases, can be obtained from $X P^{\prime} N$, where the $X$ is a noun or an adjective (including derived ones) and $P^{\prime}$ represents certain prepositions (mostly for, of) and certain verbs or adjectives depending on the pair $X, N$ (e.g. like, special to). Thus snow-shoes $\leftarrow$ shoes for snow, repeated book-burnings $\leftarrow$ repeated burnings of books, slate-grey wall $\leftarrow$ wall grey like slate $\leftarrow$ wall which is grey like slate. Many $X, N$ pairs permit the zeroing of a particular verb or preposition which is special to them: The milkman is late $\leftarrow$ The man who brings the milk is late; The milk-punch is strong $\leftarrow$ The punch contaning milk is strong and presumably not from The punch coloured like milk is strong.

Another kind of zeroing is that of I ask, I request. To obtain a common derivation for all interrogatives, both yes-no and wh-questions, it is best to take Are you coming? not from You are coming but from I ask operating on You are coming, yielding I ask whether you are coming or not. Then Who is coming? would come from I ask on the pair One is coming, Another is coming, yielding I ask whether one is coming or another is coming, which by regular and-or zeroing reduces to I ask whether one or another is coming, which reduces by wh-pronouning to I ask who is coming. In both forms of the questioning statements there is then an optional change which places a question intonation on the object of ask, dropping the whether and permuting the free subject with the following verb-
word. This yields I ask: Are you coming, or not?, I ask: Who is coming?, and further Are you coming?, I ask and Who is coming?, I ask. This whole development occurs with all verbs whose object is whether ... or . . (but not if there is a zeroed choice or the like as in I know whether he is coming or not, which there is reason to derive from I know the choice as to whether he is coming or not). E.g., He could not decide whether she was there or not $\rightarrow$ He could not decide: Was she there, or not? The connexion between question-intonation and this disjunction is supported by the fact that verbs lacking one lack the other: e.g. I doubt whether he is there, but *I doubt whether he is there or not and *I doubt: Is he there? The relevance of this derivation to our present purposes is in showing that Are you coming?, Who is coming?, is obtained by zeroing of the main verb (with its subject) whose object the question had been, namely I ask.

A similar analysis yields Please come! from I request of you: Please come! from I request of you that you please come.

There are also examples of reduction of particular words not to zero but to affixes. Thus, the relation of quietly to in a quiet manner, and the relation of considerably to to a considerable degree, would be considered merely one of paraphrase were it not that they are historically connected. The $-l y$ is a reduction of lic 'body, form' plus indirect case-ending when that was in a compound with quiet, considerable, etc. But lic plus case-ending must also have occurred as a free word, not in a compound; and there it was replaced by in a manner, to a degree, etc. Hence quietly has today the status of a reduced variant of in a quiet manner. Given this understanding, we must go further and say that adverbs of fact, such as truly, also had some preposition-noun form, such as as a true matter, which is not merely their paraphrase but their unreduced variant; for the free-word form of -ly with true must have been able to exist and must have been in effect replaced by some word which can stand freely today in this position.

A similar situation of affix-reduction and suppletion is seen in -hood. This is historically reduced from the compound position of -had 'state, condition', which in free-word position has been replaced by such words as state. Whereas book-burning is clearly from burning of books, childhood is less obviously from the state of being a child; but it is that, not merely by paraphrase, but by historical reduction in the suffix (and compound) position and suppletion is the free-word position. This derivation explains how the suffix can carry its own modifiers. Thus His early childhood was unhappy is from His early state of being a child was unhappy, with early modifying state, i.e. the eventual suffix -hood.

## 2. Amount of information

The reductions have a common property, that all of them take place on words which is one way or another contributed little or no information to their sentence.

This is the case for repeated words. Thus, since repetitional zeroing under and takes place only for words which repeat a word in corresponding position, having zero between Mozart and operas gives no less information than having wrote. Similarly for pronouns, except for degeneracy in respect to which it is the noun that is the antecedent. The indefinite nouns contribute no information. All that things tells us after read is that there is something which is being read, which we obtain anyhow if we take He read as a transitive with zeroed indefinite object, rather than as an intransitive. The which is, who is are by far the most likely words in their position, hence the least information-bearing: the only other words there, which are not zeroed, would be which becomes or the like. The be here under expect, and the various verbs zeroed for the compound, are the most expectable, given their neighbours, and are inferrable from the zeroing, and even recoverable. And the prepositions zeroed for the compound are the general ones for that relation, and very common, and hence contribute little information.

A different and interesting kind of informationalessness is seen in the case of $I$ ask, I request. While the information of the word ask is contained in the question-intonation which is imposed on its object, the subject and tense of ask do contribute extra information. However, there is one subject and tense which can be considered to contribute no information, and this happens in the performative case. For a person to say I ask: Are you coming? is the same as for him to say Are you coming?, because for the speaker to make the statement is for the question to be asked. Hence $I$ ask is zeroable. That indeed it is $I$ ask and not I asked, He asks, etc., which is zeroed here is clear not only from the meaning of the question form but also from its textual environment.

There are many other examples of reduction taking place where amount of information is low. For example, a sentence which is the object ('complement') of a verb (I promised Gohn that I would get the book) may zero its subject or object if they are the same as the subject or object of the main verb (I promised John to get the book). Which of these positions is the one that admits zeroing depends on the main verb, in a seemingly arbitrary way. But there is reason to the choice of position for zeroing. Thus, there is no referential zeroing under observe or report, etc.: I observed my slurring of the vowels, I reported his seeing me. Under prefer, admit, promise, the subject is zeroed if it is the same as the main subject: I prefer to phone fohn, I admit phoning fohn, I promised fohn to phone him. Under order or beg, the subject is zeroed if it is the same as the first object of the main verb: I ordered fohn to phone them, I begged fohn to phone me. Under deserve, suffer, undergo, the object is zeroed if it is the same as the main subject: Fohn deserves our support, John suffered their attacks. Under defend, the object is zeroed if it is the same as the first object of the main verb: I defend him from their attack.

Each one of these verb sets has a property common to its members. That
property is not their having a common meaning: it is hard to say what meaning is common to prefer and promise but not to beg or deserve. Rather, it is the likelihood that the subject, or object, under that verb be the same as the subject, or object, of the verb itself. It is reasonable to expect that the object under they undergo, they deserve should be the same they (i.e. same as the main subject), or that the subject under they admit, they promise should be the same they, and so on. But such higher likelihood means that when the main subject, or object, indeed reappears in the stated other position it contributes less information there than a different word would; and it is this low-information case that is zeroed.

A different example is seen in the reciprocal verbs. These verbs present difficulties for grammar, not least in that they require and in their subject -a rather irregular situation. They can be obtained simply if we derive, say, fohn and Mary met from fohn and Mary met each other pronouned from fohn met Mary and Mary met Yohn (which is why we do not normally have *John and his doom met, *Gohn and the brook met from fohn met his doom, fohn kept descending until he met the brook). Now, the pronouning to each other can take place with any verb: Yohn and Mary saw each other from Yohn saw Mary and Mary saw Yohn. But the zeroing of each other takes place only in verbs where given $N_{1} V N_{2}$ it is most likely that also $N_{2} V N_{1}$, so that little or no information is given there by an occurrence of and $N_{2} V N_{1}$, which is pronouned by the each other, which can then be zeroed.

Whereas high likelihood favours reduction, low likelihood can block a reduction which would otherwise take place. Thus He farms extensively has two meanings, one when reduced from to an extensive degree, and the other when reduced from in an extensive manner (as against intensive farming). But He writes extensively on this subject is only to an extensive degree. It is not that one cannot say He writes in an extensive manner, but rather that manner is not common for the pair write, extensive and therefore is not reduced there to -ly, except contrastively.

## 3. Insertion of indefinite nouns

Aside from the reductions, we must note another overt relation among sentences. This involves the extra occurrence of indefinite nouns such as that, something. If we consider such sentences as
(I) What fell is a dictionary.
(2) The dictionary is what fell.
(3) A dictionary is what I need.

We seem to have in (2) a fixed word-sequence is what inserted into a sentence; from (2) one kind of exceptional permutation yields (1) and another yields (3). But English has also
(4) That which fell is a dictionary.
(5) The dictionary is that which fell.
(6) A dictionary is that which I need.

The relative clauses here show that (4)-(6) are derived from
(7) That is a dictionary; that fell.
(8) The dictionary is that; that fell.
(9) A dictionary is that; I need that.
just like $A$ dictionary is the thing which fell $\leftarrow A$ dictionary is a thing; the thing fell, and The book which fell is a dictionary $\leftarrow$ The book is a dictionary; the book fell. Although (7)-(9) are stylistically uncomfortable, the sentences (4)-(6) imply the existence of (7)-(9) in the grammar, for there is no other efficient way of describing (4)-(6), and all other that which sentences. In turn, all sentences in the set of (1)-(3) are obtained from (4)-(6) sentences by a reduction of that which $\rightarrow$ what. The permutations which would have been needed for (1), (3) are thus not needed; they are inherent in the different orders available in (7)-(9).

## 4. Other shape changes

There are very few other changes of shape in English sentences. Almost all permutations arise from a leftward shifting of a secondary sentence into its primary. This happens especially if a noun or noun-like sequence in the second is pronouned by wh as being a repetition of one in the first, thus creating a relative clause: My friend left; I had told you about him goes both to My friend - I had told you about him - left and to My friend left, whom I had told you about $\rightarrow$ My friend, whom I had told you about, left. In the relative clause, which is, who is (and in some situations which, whom) are zeroable; certain residues then shift further to before the antecedent: The sun, which is bright, shone $\rightarrow$ The bright sun shone.

## 5. Extension beyond the attested sentences

We have seen certain kinds of reductions, mostly to zero, taking place on material that contributes little information to its sentence; and we have seen, in the is what sentences, the insertion of indefinite nouns in a way that affects the nuance of the sentence. We will now see that these reductions and insertions, in the same conditions, can be used to derive the remaining sentences of English from sentence-like formations (to be marked with $\dagger$ ) that go beyond what is normally said in English, and which will be characterized below as grammatically possible rather than actual. These grammatically possible sentences are marginal to English grammar, rather than external to it, as noted in 6 below. Their importa lies in this, that if we include them as descriptive (not always historical)
sources of the remaining English sentences, then the reductions and insertions described above turn out to suffice for deriving all English sentences from a subset of sentences (including daggered ones) having very simple structure.

A few major cases are the following:
Adverbs and subordinate conjunctions are obtained via zeroed which is on a sentence. They can be derived from predicates on the sentence, but not directly: they are predicates on a secondary repetition of the sentence, which has formed a relative clause. He addressed them quietly and He quietly addressed them are both from $\dagger$ He addressed them, which was in a quiet manner (or: quietly) $\leftarrow H e ~ a d d r e s s e d$ them; his addressing them was in a quiet manner. Similarly, He left because they phoned and Because they phoned, he left are both from He left, which was because they phoned $\leftarrow$ He left; his leaving was because they phoned. The various permutations of adverbs and subordinate clauses are precisely those permitted by zeroing of which is. The derivation via two sentences with semicolon also explains why both the verb and the adverb are each asserted in He spoke quietly. And it explains why there is no *He falsely left, *He left improbably, although one can say That he left is false, That he left is improbable, and of course, He truly left, He left probably. The reason is that the source would be *He left; that he left is false and *He left; that he left is improbable, and it is the conjunction of the two which is rejected in each case.

English, like other languages, has various words of peculiar and apparently multiple syntactic statuses. It turns out that for each such word, its various uses can be obtained from a single source, by means of 'expectable-word' zeroing such as has been seen above. Consider, for example, only, except, but. Because of their concessive meaning, a favoured (expectable) first sentence under them is one that contains everyone else (anyone else) and is otherwise the negation of the second sentence under them. Thus, for but: Mary cooked, but fohn was late is possible, while Mary cooked, but Gohn didn't (cook) is more comfortable, and the distinguished case is seen in Everyone else cooked, but Mary did not (cook). Now we first consider the zeroings under only. We seem to have two syntactic statuses for only: (1) I spoke, only fohn wouldn't listen (with conjunctional only) and (2) Only fohn wouldn't listen. If we start, in the conjunctional type (i), with Everyone listened, only $\mathfrak{F o h n}$ wouldn't listen, the first sentence is the favoured one and zeroable, yielding Only fohn wouldn't listen, which is of type (2) above. The meaning of only in type (2) is precisely the sum of that in type (1) plus the expectable sentence Everyone (else) listened. In (2) it seems as though only carries this as a covert or implicit meaning, but actually this is merely the meaning of the zeroed sentence. As to how it is that the conjunction and can occur before the conjunction only: if we start with We argued and everyone listened only fohn did not listen we obtain We argued and only fohn did not listen.
As to except: it is an ordinary conjunction in fohn invited her, except that she
couldn't come. In fohn agreed, except Mary did not agree, regular zeroing yields John agreed, except not Mary. In the favoured everyone + not case, as above, with not being said in the second sentence, the everyone-sentence is not zeroable, but the not is: from Everyone (else) agreed, except Mary did not agree we have, by repeated-word zeroing, Everyone agreed, except not Mary, and by expectableword zeroing on not, Everyone agreed except Mary. It is in this way that the sentence with not comes out as a paraphrase of the one without not. Similarly for but: First, John invited her but she couldn't come. From Everyone (else) agreed but Mary did not agree we obtain Everyone, but not Mary, agreed and Everyone but Mary agreed with the same expectable-word zeroing of not. However, but permits also the expectable-sentence zeroing seen under only. Hence Everyone (else) agreed, but Mary did not agree reduces to the rather rare But Mary did not agree with stressed but in the sense of only. It is in this way that we reach the paraphrase between There isn't but one glass left and There is but one glass left, both from There isn't anything (else) left, but there is one glass left, with and without zeroing of not. In this way too the apparently different uses of but as conjunction and as synonym of only are obtained from the same source.

Another case of zeroing which goes beyond the obvious is seen when we assume that any assertional sentence, including such as become components of larger ones, have a zeroed I say, in a way comparable to the zeroing of $I$ ask, I request. E.g. I say that he is here $\rightarrow I$ say: He is here. $\rightarrow$ He is here, I say. $\rightarrow$ He is here. There are many indications that any sentence can carry a zeroable $I$ say: e.g. interpellations as in He is uncertain, not to say confused, where the subject of say is presumably $I$, zeroed from the first $I$ say in $\dagger I$ say that he is uncertain, if it is not for me to say that he is confused. A simpler situation is He's home, because his car is in the garage where because makes sense only if I say was present: I say he's home because his car is in the garage. Various problems, such as those in direct and indirect discourse, are simplified by this eminently possible, if uncustomary, analysis.

A related analysis can apply to the tenses. Tenses are peculiar in that their normal definition is metalinguistic, referring to the time of the speaking of the sentence. However, there is also evidence that tense indicates the time-order between verbs in the sentence - more clearly in some other languages than in English. It is possible to obtain these and various other properties of English tenses by starting with, for example,
$\dagger$ John expects Mary's coming; Mary's coming is after John's expecting it.
$\rightarrow \dagger$ John expects Mary's coming, which is after John's expecting it.
Here fohn's expecting it is zeroable, as being a repetition:
$\rightarrow$ John expects Mary's coming, after.
Here we can posit that after imposes will on Mary's coming, much as ask imposes
question intonation on its object-sentence (in I ask: Are you coming?). Then we obtain

John expects that Mary will come.
The tense on the main verb comes similarly from its time-order to the I say on it:
$\dagger$ I say that John expects that Mary will come; John's expecting that Mary will come is before my saying it.
$\rightarrow \dagger$ I say that John expects that Mary will come, before my saying it.
$\rightarrow$ I say that John expected that Mary would come.
$\rightarrow$ I say: John expected that Mary would come.
$\rightarrow$ John expected that that Mary would come.
Note that the -ed on expect also attaches to verbs which had been previously tensed in respect to expect: will come to would come. This is a roundabout way of obtaining tense; but, as with the pronouns, it saves tense from being sui generis in grammar, by deriving it from ordinary constructions by means of known reductions.

Like reductions, the insertion of indefinite nouns, too, explains difficult grammatical constructions. For example, there is the difference between nonrestrictive (or 'descriptive') relative clauses, as in

A man, whom I know, phoned
and the comma-less restrictive
A man whom I know phoned;
in the restrictive form, phoning is asserted about someone identified as a man I know. The form and special meaning of the restrictive can be readily obtained if we start with
$\dagger$ Someone phoned; the same someone is a man; I know the man.
$\rightarrow$ Someone phoned; the same someone is a man, whom I know.
$\rightarrow$ Someone, who is a man whom I know, phoned.
where the earlier relative clause loses its comma when compressed into a further relative clause. Then, zeroing the indefinite someone and the following who is, we reach

A man whom I know phoned.
The special restrictive meaning comes from the fact that phoned does not enter into a grammatical construction with man until after man had received the modifier whom I know. In contrast, the non-restrictive case comes from

A man phoned; I know the man
$\rightarrow$ A man phoned, whom I know
$\rightarrow$ A man, whom I know, phoned,
where phoning is asserted of the man independently of my knowing him.
If we accept set, team, or the like as indefinite nouns, then the same insertion enables us to derive and between nouns from and between sentences, and to make the distinction between, e.g., Gilbert and Sullivan wrote operettas and Mozart and Beethoven wrote operas. We take the former from

A team wrote operettas; the team contains Gilbert and the team contains Sullivan
$\rightarrow$ A team which contains Gilbert and Sullivan wrote operettas,
where a team which contains (or: is of) is indefinite-noun plus expectable-word zeroing, like something which is. This analysis also resolves the difficulties with collective verbs, such as gather.

As a final example of a transformation, consider the passive, which was the first to have been established. It is clear that the passive is not simply an independent rearrangement and form-change of the active, since all its ingredients are otherwise known in the grammar, in closely related uses. The by plus subject occurs in nominalizations: The chopping of trees by settlers denuded the forest. The -en/-ed occurs as an affix having approximately the same semantic and descriptive (though not historical) relation to some such completive word as state, as does -hood in the childhood example. This is seen in the very early -ed on nouns (moneyed, hard-headed), which descriptively can be considered to be the same morpheme as -en|-ed, and which means 'state' as in in the state of having money, etc. It is also seen in the development of the perfect, whose early stage was, e.g., I have the book closed, i.e. $\dagger$ I have the book in the state of my closing of it; note that I have closed the book means that it is I who closed it, and lacks passive connotation, so that the source is not $\dagger I$ have the book in closed state. In a parallel way we have the passive: The book was found by fohn from $\dagger$ The book was in the state of the finding of the book by $\mathfrak{F o h n}$, with of the book zeroed as repetition. This derivation is equally hospitable to agent-less passives such as A kitten is born with its eyes closed (although nobody closes them) from $\dagger A$ kitten is born in the state of its eyes' closing (or: of the closing of its eyes). Similarly for intransitives, where the resultants are only statives and not passive at all: as in $W e$ are agreed from $\dagger W e$ are in the state of our agreeing; The sun is risen from $\dagger$ The sun is in the state of its rising. The completive aspect of the passive thus does not have to be separately explained. It arises from the character of the -en/-ed (and of its free-word variant state); and the extension of the passive to the non-completive, as in It is being built is quite recent. The subject of the passive is thus not a permuted object of the verb; rather, it is the subject of is in a state, in reference to which the object of the verb is zeroed. This analysis
does not require that state be a free-word source for -en/-ed, but only that -en/-ed be an operator like in a state ( 9 below). Of course, non-periphrastic passives, in some other languages, would be analysed differently.

## 6. The status of the daggered forms

The daggered sentences have the property that they differ from attested sentences of the language only by the same listable changes (made in the same conditions) as hold among attested sentences. Furthermore they consist of English words appearing in positions which those words have in the attested sentences of English. No clear line can be drawn between them and the normal sentences; indeed, already the sources of the that which is sentences could have been daggered, even though they are inescapable as being the relevant sources there. Note also that the set of attested sentences is not well defined. Many sentences are marginal, or used by some speakers or writers and not others. Some of the daggered sentences are grating to the ear without being excludable from the grammar in any principled way.

The relation of the daggered sentences to their normal resultants is not always historical. True, -hood is the reduced compound form of a word replaced by state, and thus suppletive to it. But we cannot say this of een/-ed. Nevertheless, descriptively, the detailed conditions and meanings in which -en|-ed occurs are as though it were reduced from some such word as state. Or rather, -en/-ed retains, in its suffix position, properties that belong to a word such as state. Just as we saw in early childhood the word early as a modifier on -hood, so in e.g the freshly isolated heart (from a medical paper), freshly, which is not an adverb on to isolate, appears as adverb on the -en/-ed: the heart freshly in the state of the isolating of it.

We therefore take these daggered sentences as constituting, together with the normal sentences, an extended set of grammatically possible sentences.

## 7. The entry and reduction system

The extended set of grammatically possible sentences has mathematical and linguistic properties. If we form in that set a subset $\{I\}$ consisting of only the unreduced sentences, whether daggered or not, we find that they have a transparent structure, describable as follows:

In the sentences of $\{I\}$ we define a relation $\{Y\}>\{X\}(\{Y\}$ is later than $\{X\} ;\{X\}$ is prior to $\{Y\}$ ), where $\{X\},\{Y\}$ are sets of simple (generally affixless) words entering the sentence. The relation holds for just those cases where a word $Y$ of set $\{Y\}$ is present in $\{I\}$-sentences only provided that some word $X$ of set $\{X\}$ is present there : we say that $Y$ requires $X$. Thus every $\{I\}$-sentence containing probable must also contain a word of the set (verbs, etc.) which
includes come, fall, large (His coming is probable, but not * He is probable); every $\{I\}$ - sentence containing fall must contain a word of the set of $\mathfrak{F o h n}$, rock, etc., but not necessarily a word of the set of come, probable, etc. (e.g. Fohn fell, which contains no other verb). (In the second case, we could say that rock requires fall, rather than fall requires rock; but fall is morphologically and positionally similar to probable and to the other words which have non-null requirements, whereas rock is not.)

The entry relation is a partial order: If $Y>X, Z$, it may be that $X>Z$, or $Z>X$, or neither. This last case we see in fohn wears hats, where wears $>\mathcal{F}$ ohn, hats, but $\mathfrak{f o h} n \ngtr$ hats, and hats $>$ Fohn. If, for word-occurrences $Y, X, A$ in an $\{I\}$ - sentence, $Y>X$, and there is no $A$ such that $Y>A>X$, we say that $Y$ in that sentence is the operator, or next later entry, on $X$, and $X$ is an argument, or immediately prior entry, of $Y$; also that $X$ was free for $Y$. In the linear form of the sentence, the operator appears immediately after the first of its arguments, as in $\mathfrak{F}$ ohn wears hats; if we accept some suffixes as operators, these appear after their second argument: in John was seen by Mary the arguments of -en/-ed are Yohn, see, and those of see are Mary, fohn.

It is found that each word is associated with particular ordered word sets, members of which must be its immediately prior entries; we call these sets the argument requirement (or entry requirement) for that word: e.g. the requirement for wear is two words of the John, hats class. To avoid infinite regress or circularity, there must be some words whose requirement is null (called primitive arguments, $N$ ), some whose requirement consists only of primitive arguments (these are called elementary operators), and some whose requirements include operators (these are the non-elementary operators). Operators are written with subscripts to indicate their ordered arguments: elementary operators include $O_{n}$ (e.g. sleep in fohn slept), $O_{n n}$ (e.g. wear); non-elementary operators include $O_{o}$ (e.g. continue, probable in Fohn's playing continued, That fohn is late is probable), $O_{n o}$ (e.g. know in fohn knows that she is late), $O_{o n}$ (e.g. surprise in fohn's being late surprised me), $O_{o o}$ (e.g. cause in John's phoning caused my return), $o_{n n o}$ (e.g. tell in $I$ told John that she left), $O_{\text {noo }}$ (e.g. attribute in $I$ attribute her return to $\mathfrak{F o h n ' s}$ phoning). It is found that the only word-classes needed for arguments are $N$ and $O$ : i.e. an argument of a word is either null, or it is any primitive argument, or it is any operator of whatever kind. For example, there is no verb whose object must be an elementary operator such as sleep and not a non-elementary one such as know: If we can say I think that he slept, we can say I think that he knows that she came.

Every operator forms with its arguments an $\{I\}$ - sentence, and appears there after the first one of its arguments, as in fohn wears hats. Every $\{I\}$-sentence is formed out of $N$ and operators belonging to some ten subsets - the entry-classes $O_{n}, O_{n n}, O_{o}$, etc. - with no other factors entering into its construction. There are
few or no grammatical restrictions on the members of an argument set, aside from graduated unlikelihood due to nonsensicality. Thus if wear requires two $N$, it is not necessary for the first to be animate and the second clothing. If we consider metaphor, fairy-tales, and arrant nonsense, there is no primitive argument $N$ which can be absolutely excluded grammatically from either argument-position.

The reductions, if they are made, are made on (or in respect to) words entering a sentence at the moment at which they enter. They cannot normally be made after further entries have come in. This fact is the source of the nesting of grammatical constructions, e.g. the fact that later adverbs are farther from the verb than earlier ones (except for intrusion with commas). Also, the determination of whether a word is contributing low information, and is thus reducible, can be made on the basis of the relation of the word to its prior (in most cases immediately prior) or next later entries. If come is zeroable under expect, and manner rejects reduction to -ly when between write and extensive, it is all on the basis of their likelihoods to their next entries - to expect, and to write, extensive without regard to the rest of the sentence.
These conditions on entry and reduction suffice as grammatical processes. The grammatical patterns that emerge for each language are due to the specific types of entry-classes and reductions, and to the similarities of physical shape and of domain among the various reductions. Given a list of the words in each entry class, $N, O_{n}$, etc., and a list of the specific reductions with the subdomains to which they apply, these processes produce the actual sentences of the language.

## 8. Mathematical properties

The set of grammatically possible sentences is closed under the entry-order relation, making it amenable to mathematical investigation. Furthermore, the objects which participate in the entry order are defined solely in terms of that order: a word is defined as having entry requirements of $N$ or $O$ where $N, O$ are defined by their having either null or non-null entry requirement; this makes the set formed by the entry-order relation a mathematical object. The further grammatical events, mainly the reductions and the insertions of indefinites, are defined on likelihood and other relations among the participants in the entry-order relation.

The entities in the set which is closed with respect to this entry-order relation are sentential word-sequences. But they could be anything else, and if these other things had the same estimated likelihoods of entry that the words do, then the new entries would constitute under the entry-order relation a set isomorphic to the set of sentences, as indeed we have in writing vis-à-vis speech. In contrast, concatenation as an operation in the set of all word-sequences, or manipulations of word-position or the like without regard to entry status, does not distinguish
sentences from non-sentences without a large ad hoc apparatus of grammatical conditions.

The grammatical analysis of a sentence consists in finding what ordered entries and reductions, the occurrence-conditions for each being satisfied by its precursor, yield the sentence; it constitutes a decomposition of the sentence into a canonical form.

Abstractly, therefore, we are dealing here not with words defined as having certain meanings and certain morphological structures, and not with a body of rules stated on particular words or combinations, but with the set of grammatically possible sentences as a mathematical object. As a result of all this, it is possible to state various relations among the entities, and the subsets, in the set of sentences. And since the interpretation of the mathematical object is a real thing, language, the subsets and relations have real interpretations too. Only a brief sketch of some subsidiary relations can be given here.

We take the set $\{S\}$ of grammatically possible sentences with homonymous (ambiguous) sentences, which are reduced from different sources, being taken as different sentences. In $\{S\}$, each operator which has precisely one operator in its argument-requirements (called unary) maps the set $\{S\}$ into itself, i.e. onto the subset whose latest operator is that unary. Each binary operator, which has two operators in its argument-requirement, maps $\{S \times S\}$ into $\{S\}$. Each reduction maps a subset of $\{S\}$ (having a particular latest or penultimate entry) onto a subset (having that reduction). All of these are thus transformations and partial transformations of the set $\{S\}$, and it is for this reason that these intersentence relations were called transformations.

In $\{S\}$, the subset $\{I\}$ of sentences containing no reduction is closed with respect to the entry-order relation, and can be called a sublanguage. In $n$ successive reductions in a sentence, the domain of the $m$ th ( $m \leqslant n$ ) cannot be larger than that of the $m-\mathrm{I}$ th (since the $m$ th is defined on the resultant of the $m-\mathrm{Ith}$ ); hence while it may be that there are some unreduced sentences which have no reduced one as image, there are no reduced sentences that lack an inverse image in the sublanguage. This, plus the fact that the reductions do not alter the information in the sentence, means that all the information in the reduced sentences is present in their inverse images, i.e. in the sublanguage. Hence the set $\{S\}$ contains a sublanguage, $\{I\}$, whose sentences have the transparent entry-order construction, unrestricted by reductions, and contain all the information in the sentences of the language.

Various algebraic structures can be defined in $\{S\}$. Equivalence relations can be defined in $\{S\}$, yielding various partitions of the set. The set $\{S\}$ is a semigroup under and as binary composition. If we take two sentences in it to be in the same equivalence class $E_{i}$ when they contain the same ordered unary operators and reductions, then and provides a binary composition in the set $\{E\}$ of equivalence classes, with $\left(E_{A}\right.$ and $\left.E_{B}\right)=\left(E_{A}\right.$ and $\left.B\right)$. The elementary sentences of the
language are reachable via the kernel of the natural mapping of $\{S\}$ onto its quotient set $\{E\}$; it is for this reason that they were called kernel-sentences. (Cf. fn. 61 of Harris (1957) and more generally on transformations (Harris, 1970, 1968).) Another partition puts into one equivalence class all sentences having the same partially ordered entries: all sentences in an equivalence class are then paraphrases, and one is unreduced, i.e. a member of the informational sublanguage above.

Defining the reductions on entries rather than on sentences makes possible a finitary grammar: a finite set of elements and operations, and finite domains (prior entries, not whole sentences) for these.

## 9. Properties of language

This system of grammar can be presented as a set of formation and transformation rules on an alphabet consisting of $N$ and $O$. Given which words are in each entry class and which are in the domain of each restricted reduction, this system provides not merely the grammar of sentence structures but the grammar of actual sentences - the indication of which word-sequences are sentences, although one would still need information about likelihoods and about which reductions are common. In general, the domains of the restricted reductions require listing, at least for borderline cases, even though they are not arbitrary and must be related to extra-high likelihood (yielding zeroing), extra-low likelihood (semantic rejection of a combination), and the like. Analogic and other processes on the resultants of entry-and-reduction may lead to the spread of certain constructions and grammatical patterns.
The theory tells what can be found in a sentence and where it can be found: reductions apart, every sentence must contain at least one primitive argument $N$, and it must contain as many $N$-argument positions under its operators as it contains $N$ (before reduction). And for every operator or operator-pair in it, it may contain an operator with one $O$ or two $O$ respectively as arguments. Since the position of each operator is immediately after the first of its ordered arguments, and since each reduction leaves a trace (as follows from the recoverability of zeroed words), with the main permutation being just the secondary-sentence shift, the possible locations of the entries and their possible physical shapes are known. An effective decision procedure can be formulated for analysing every sentence into ordered reductions and entries. There are, of course, considerable complexities due to degeneracies of reduction, many of which are only local and can be resolved by reference to other entities in the sentence.

The sentence analysis presented here reveals the nature of grammatical transformations. (I) Some transformations are simply reductions, as in He reads by zeroing of the indefinite appropriate object in He reads things; or a succession of reductions as in His childhood was happy from For one to be happy is a state;
his being a child was that state. (2) Other grammatical transformations are reduced - or as though reduced - from added entries which had very general meanings, as in the passive -en, where The trees are in a state of the chopping of the trees by settlers first zeroes of the trees, then reduces in a state of the chopping to chopped, so that it may seem that the object of chop has moved to subject position while chop has somehow changed to is chopped. (3) Yet other transformations are the results of distributing the words of a sentence through two sentences, filling in the remaining positions with non-specifying words (e.g. demonstrative pronouns). The non-specifying words are then reduced or zeroed, but their presence leaves a nuance on the sentence; as in What fell is the book, or in the restrictive relative clause. What is common to all of these is that the words of the 'source' sentence appear in the same or equivalent grammatical relations in the transform, and that the change can be made in all or many sentences of the source form. The reason for this is that the added words in the expanded source forms are of such general meaning (state, that, something) that they do not affect the word likelihoods (selections), and contribute only a nuance of meaning, and that they therefore invite reduction to affixes or the like which can then be considered 'constants' of the transformation. And indeed the transformations which are pure reduction (I) are pure paraphrases. The others $(2,3)$ have a difference in nuance, and this difference is visibly produced in the source form, by state, that, etc. in their given positions, and is thereafter paraphrastically preserved in the reduction. It is thus seen that transformations are not a set of word-manipulations coming full-blown, and not a grammatical process at all, but an effect. They are the effect of reductions and of broadselection entries. The only grammatical transformations which are found in a language are those that can be reached by such entries and by the known reductions.

The method of analysing a sentence $A$ is to look for a source which (1) is made out of words that have entered the source solely on entry-order grounds, and $(2)$ is reducible to $A$ by attested reductions in attested conditions. 'Solely' means that any morphological structure the word may have is not used additionally to its entry-order in accounting for the position of the word. Indeed, we may accept that certain operators are affixes (e.g. -en/-ed) entering into affixal position upon their second argument.

The different elements of language structure in terms of the present theory fit the different universalities of linguistic features. The system of argumentrequirements and of differences in relative likelihood (selection) for words in a single entry class seem to be universal, although some languages, for example, have few primitive arguments (concrete nouns and demonstrative pronouns). The main types of reduction seem to be found everywhere - referential zeroing and pronouning, and some kinds of 'expectable' zeroing. Indeed, the intertranslatability of language relates in large part to the similarity of entry-structures and
of the main reduction types, with $O_{n n}$ operators in one language being translated largely by $O_{n n}$ operators in the other, and so on. The specific reductions are more nearly language-specific, and in several cases the most special and difficult-to-analyse transformations are peculiar to the one language in which they are found.

The special grammatical patterns of each language or family of languages arise largely from characteristics of the reductions: either from special shapes (such as the compound-word stress), or from similarities in shapes (such as the similarity of noun-modifiers to the subject and object of a nominalized sentence), or from having similar domains for various reductions (this creates such grammatical categories as nouns), or from complementary domains (which create conjugations and the like)-- and finally from the frequency of use of some reductions (as in the auxiliaries). But these patterns may be affected also by analogic extensions of sentence features, by systematic borrowing, etc.

Many of the source forms reached in this analysis reconstruct known or possible historical sources. There are cases of a word moving to a new grammatical status, replacing its previous occupant, as in will becoming a future tense, and in the compound (periphrastic) tenses of various European languages. But many other extensions and specializations of use of a word do not get as far away from their earlier uses. They may differ locally, as in the case of the auxiliaries, but in their farther environment or in their complementary words (variants) they may still belong to their class of origin: can can still be considered a variant of a tensed $O_{n o}$ verb. And the development of really new grammatical relations is rare and slow indeed. The status of modifier may seem to promise such a development; but there are many transformational connexions which keep the set of all modifiers closely tied to being an entry-and-reduction resultant, namely to being residues of a secondary sentence one of whose arguments is the same as one of those in the primary sentence.

There are, however, other situations which exhibit not so much the tenacity of history as the inherent limitations of what language can or cannot express. Thus morphology may seem to present a degree of freedom independent of syntax, one which would make morphology-rich languages able to say things that morphology-less languages cannot. This turns out not to be the case (witness their intertranslatability, for one thing). The affixes are equivalent to reductions of operators, reaching their affix status by established reductions. This may be due to the fact that many of them were indeed such operators. For the others, granted that they developed outside syntax, they were domesticated, upon entering a sentence, by the entry relation which creates syntax. Whatever can be expressed with the aid of morphology can be expressed (given the availability of the words) without morphology, by operator-argument relations among simple words, as though the affixes were reductions of such words.

Since the entry-structure of language is so simple, the bulk of what is called
grammar is created by the reductions. But the reductions do not alter the entrycontent of sentences. In all cases, allowing for degeneracies, when an entry has been zeroed, its presence can be reconstructed from the structure of the sentence in its post-zeroing form, so that we do not even have to say that the word has been dropped but only that its phonemic shape has been changed to zero. If the zeroing is referential we know (up to ambiguity) from the antecedent which word has been zeroed. If the zeroing is of a non-specific or of an expectable word, its meaning-contribution to the sentence was small and can be found in the new grammatical relation that results (e.g. the verb-less milkman, the object-less He reads). This means also that if a sentence contains covert meanings, not explicitly contained in any of its words, these are due to zeroed words in the source form.

The result of this is that the meaning of each entry, and the meaning of its operator and argument relation to prior and next entries, is preserved in the sentence as it grows with new entries and as it is transformed. A sentence is a particular ordering of entries, and its meaning, the meaning of the ordering of the entries, is the ordering of the meaning of the entries. In making a syntactic analysis we therefore obtain the meaning of the sentence directly in terms of its entering words; and the components of the syntax are the components of the meaning. The present method enables us to reconstruct a sentence in such a way that no component says anything which is not said in the final sentence. Thus No man came is derived not from $A$ man came; the man is none but from I deny that a man came. And Frozen food lasts long is taken not from Food lasts long; that food is frozen but from Something lasts long; that something is food; that food is frozen.

Since even the nuances brought in by transformations are brought in by zeroable words, this means that all the information expressed in the sentences of the language is expressed in the subset consisting of their unreduced, possibly reconstructed, source sentences which have only the entry structure. And since the entry relation has little or no grammatical restriction on the words of an entry class, but only differences in likelihood, this means that the information in language is expressible in these unrestricted entry constructions. Thus it is not any complexities of the real world of information that are responsible for the complexities so characteristic of grammar. For the information alone, a very simple system, the entry-order, suffices.

One sees from the entry and reduction analysis that the structure of language is an information-carrying system; if we take its discreteness into account it is even better understandable as an information-transmission system. If we exclude gestural, non-grammatical, features such as intonations of irony, language has no structural provision for the expression of feeling; one only makes statements, possibly false, that one has this or another feeling. The use of language as a vehicle for feeling and art, as in poetry, is done by a secondary
manipulation of an existing language and grammar. And where the structure of music, certainly a vehicle for feeling and art, includes such elements as sequences of notes and variations defined on the sequence, the word-sequences of language, as the symbol-sequences of mathematics, are only projections of the partially ordered entries.

The informational character of language comes out even more sharply in the specialized languages of individual sciences, where one can see not only a somewhat different grammar, but even something new: a somewhat different type of grammar, made possible by the consistency of subject-matter and by the fact that whereas language has no external metalanguage, the sublanguage of a science can draw upon the whole language for its metalanguage.

From the nature of the separate structural features which together create language, one can see a naturally growing instrument for transmitting information. The entry requirement is an adequate form for predication, which is seen, e.g. in the derivation of the interrogative, to be the source informationdevice for all language; if a word of class $Y$ is said only provided one has said a word of class $X$, then $Y$ exists in sentences only as something said on $X$ - as close a structure for predicating $Y$ of $X$ as one might expect.

Language is also reasonably efficient when looked upon as a naturally developed instrument for transmitting information. First, it is the least structure needed for the entry relation: the arguments are only $N, O$, and not arbitrary subclasses of $O$, or even $O_{n}$ as vs. $O_{o}$, etc. And the number of arguments goes up to only two or three - longer arguments could always be paraphrased by operators on operators. Second, the reductions do not take place upon sentences as a whole (e.g. by judging their redundancy) but upon the entering words (by judging their informational contribution in respect to each other). Third, the reductions make for a great shortening of the sentence without changing its information. The degeneracies which result from some of them could have been avoided by having different reductional shapes. That they do not avoid this is as much a result of the unplanned character of language as are the homonyms that result from certain sound changes.

One item of efficiency, or of unplannedness, is that many reductions have become rules rather than preferences. For example, the difference between verbs and adjectives can be stated as the tense being attached directly in the case of the less durative operators (verbs), and via a carrier be in the case of the more durative operators (adjectives). Borderline cases could have remained a matter of preference, depending either upon the speaker or upon the meaning in the context: e.g. sleep, ill could have been sometimes said as verbs and sometimes as adjectives. Instead, although different languages make different decisions about such words, within a language each word is fixed - sleep a verb, ill an adjective - so that to use ill with the less-durative, verb-like, placing of tense constitutes not merely a delicate variation in meaning but an outright 'mistake',
a flouting of convention, albeit a very old convention, and not not maintained by institutional interests. Whether one sees this as efficiency or as an unnecessary restriction upon freedom of action may depend upon one's views of what is desirable in society.

Given all this, the structure of language can be seen as an outgrowth of the uses in which language was developed. There is no need to appeal to some independently existing structure or structuralism - whose existence before language or independent of language could in any case not be explained - to see how the structure of language came to be what it is.

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[^0]:    [r] Given as a lecture in Somerville College, Oxford, 16 March 1977.

