

I. A THEORY OF LANGUAGE STRUCTURE

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THIS paper presents a theory of the structure and information of sentences, which consists of three fundamental relations: the ordered entry of words into the making of a sentence; inequalities of likelihood of occurrence of a particular *n*th entering word in a sentence in respect to the choices of words entering immediately before the *n*th; and the reduction in shape of certain words which have, upon their entry into a particular sentence, exceptionally high likelihood of occurrence, i.e. low amount of information. The first two relations determine the structure of a base subset of sentences which have a transparent grammatical structure, and contain little or no grammatical restrictions, and carry all the substantive information carried in any sentence of the language. The last relation is in general optional, and does not materially change the information in a sentence. It introduces a secondary grammatical complexity into sentences, and introduces restrictions, subsets, and grammatical rules into the language. From the base sentences all the remaining sentences of the language can be obtained by a direct application of the stated set of reductions, in a manner suitable to the constructing and, with certain short-cuts, to analyzing of sentences.

The adequacy of these fundamental relations to characterize the sentences of the language, and the simple nature of the relations themselves, suggests that the structure of language is relatable to the conditions of language's existence without appeal to some independently existing structuralism. It may be philosophically relevant that the structure and semantic relations of sentences can be obtained from this model, and that the sentence types which are involved in various philosophical issues can be identified in particular ways as cases of these general structural relations.¹

I. OPERATORS

1. *Entry Order*

The theory starts from a relation between word-occurrences in a sentence, when the juxtaposition of one word to another is sufficient to constitute a sentence. It is similar to the relation noted in categorial grammar;² but the use made of it here is different. The relation is clearest when we consider that many sentences contain sentences, in altered or unaltered form, together with word-sequences that are not themselves sentences. For example, the sentences *Mary phoned*, *John arrived* are contained in: *Mary's phoning ceased*,³ *John denies that Mary phoned*, *Mary's phoning entailed John's arriving*, *Mary phoned and John arrived*. To treat this relation, we introduce a term: If the characterization of *B* involves reference to *A*, but the characterization of *A* does not involve reference to *B*, *A* is called descriptively prior to *B*, and *B* descriptively later than *A*. In the sentences above, *Mary phoned* is descriptively prior to *Mary's phoning ceased*. The only status that *ceased* has in respect to the set of sentences is that it constitutes the increment between certain descriptively prior sentences such as *Mary phoned* and corresponding later sentences such as *Mary's phoning ceased*. We therefore say that in the latter sentence, *ceased* enters after the included sentence *Mary phoned*, and that *ceased* is the operator (or later entry) on *Mary phoned*, with *Mary phoned* as its argument (or prior entry). Similarly, *entailed* and *and* are operators on two arguments: on the sentence-pair *Mary phoned*, *John arrived*.

We next consider sentences which contain no sentence as a proper part of them, and where we cannot say that one part is descriptively prior to another. In *Mary phoned*, the word *phoned* has the same morphology as *ceased*, *entailed* (though not as *and*), i.e. it receives tenses and certain other affixes

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² See Kazimierz Ajdukiewicz, "Die syntaktische Konnexität," *Studia Philosophica* 1.1-27 (Syntactic Connexion, pp. 207-231, in Storrs McCall, (ed.), *Polish Logic 1920-1939*, Oxford 1967); Joachim Lambek, "The mathematics of sentence structure," *American Mathematical Monthly*, vol. 65 (1958), pp. 154-170; Henry Hiż, "Grammar Logicism," *The Monist*, vol. 51 (1967), pp. 110-127; "Computable and uncomputable elements of syntax," *Logic, Methodology and Philosophy of Science III*, ed. by B. van Rootselaar and J. F. Staal (Amsterdam, 1968), pp. 239-254.

³ The alteration by . . . 's . . . ing, and the insertion of *that* here, are introduced as indicators of argumenthood in 1.1, end.

ment is a pair or triple of argument-words impose a preposition as argument-mark on their non-first argument-words: *John relies on Frank, John attributes this to Frank, John attributes this to her having phoned*. In some languages the elementary arguments (sect. 2) of an operator receive suffixes (called case-endings) that indicate their argument order: nominative for first argument-word, dative for second or third argument-word of certain operators, etc.

2. Argument-Requirement Sets

We next consider the possibility of classifying words in respect to their entry into discourses. If we ask about the likelihood of occurrence of individual words, we find that each word, in entering as operator, has a unique gradation in the likelihood of its occurrence, in respect to the various words which can appear in a given argument position under it. Despite the uniqueness and gradation, a classification of words in terms of likelihood is achievable if we distinguish zero likelihood from non-zero. Thus, *slept* has in its lone argument position *John, dog* (*John slept, The dog slept*), less frequently *fish*, still less *tree* (as in *The trees slept every winter until the sap began to flow*), more rarely but not impossibly *bacteria, virus, rock* (as in *The bacteria slept because of the low temperature, These rocks slept here through the ages*), but presumably never (except in linguistic discussion) *because, go* (as in *John's returning because we phoned slept, John's going slept*). And *entails* has in its two argument positions the pairs *return, leave* and *win, move* (as in *My winning entails our moving*) but not *he, me* (*John entails me*). We can now define as the *n*th place argument-requirement (or argument-demand) of a given operator the set of all words which have non-zero likelihoods of occurring in the *n*th argument position of that operator. Thus the first-place argument-demand of *slept* includes *John, dog, fish, tree, bacteria, virus, rock*, but not *because, go*; the second-place argument-demand of *entails* includes *leave, move*, but not *me*. The whole argument-demand of an operator which has *n* arguments is the set of ordered *n*-tuples of its arguments in the *n* positions: for *entails*, it includes *return, leave* and *win, move*, but not *he, me*.

We now find that only two word-sets appear in argument-demands, for any of the operators: The whole set of operators, *O*, on whatever arguments;

and the set of words, *N*, which are not operators on any arguments.⁵ Thus, the argument requirement of *slept* is the set of all words which are not themselves operators (except as in fn. 5), and the argument requirement of *entails* is the set of all pairs of operators.

To show that argument demands are indeed only *N* or *O*:

(1) There are operators which have both *N* and *O* in one of their argument positions (e.g. *John caused this* and *John's phoning caused this*); and there are words which have arguments in certain occurrences but not in others (e.g. *The paper tore, He papered the walls*). In such cases, one form can be shown to consist of the other plus reductions (section II). There are also words which have arguments in certain occurrences and not in others, without satisfying the conditions for one form to be a reduction of the other. In English, these cases are not regular, and can best be understood as a word appearing in two sets: e.g. *The rock fell, The boat rocked; He ate a prune, They prune the trees*.

(2) Operators are not restricted to occurring on proper subsets of *N* or *O*, with the possible exception in one language or another of a few restricted subsets which would have to be listed. Thus *slept* is not confined to arguments naming animals or the like. Its use with other arguments is not a matter merely of metaphor, but also of extended and marginal meanings: a horticulturalist may consider that his trees indeed sleep the winter and need a particular amount of sleep. In addition, there is the use of words in altered perceptions of the world, whether serious or in speculative and imaginative writing: a story might have a house speaking to its inhabitants, going to sleep, etc. True, the house is then seen as animated, but this does not change the fact that *The house slept* then occurs, and not even in a metaphoric sense. There is no member of *N* which can be assured of not occurring as argument of *slept*. Even *The vacuum slept* might occur as a sentence, say in a child's cosmology book, in a way that *Go the a* would not.

From this there follows: There must exist a certain subset of words, or word occurrences in sentences, that have no arguments, since the first other word (outside this subset) to enter a sentence can do so only if an argument-less word occurs in the sen-

⁵ There are in addition two listable sets of words in each language: (1) words which are shown in the present theory to be reduced forms of particular operators or of word-sequences constructed by the operator-argument relation described here: e.g. *the*; (2) words which do not enter into grammatical combinations, i.e. are arguments only of metalinguistic operators (sect. 4): e.g. *hello, ouch* (in *He said: Hello.*). These last are a limiting case of the sentence-making entry-order in that they can be considered as constituting a sentence by themselves.

tence (even if that word is later zeroed, sects. 6–8). These argument-less words are the elementary arguments, N , such as *Mary*, *rock*, *virus*. In addition, if a language has any words other than elementary arguments—and it must if it is to have sentences—it must have some words, or word-occurrences, whose arguments are only N , since a word any of whose arguments is itself an operator could not enter a sentence whose only prior entries are N . These are the elementary operators: O_n (*slept*), O_{nn} (*ate*), etc. In addition, a language can have also words, or word-occurrences, some of whose arguments are themselves operators. These are the non-elementary operators: O_o (*continued*), O_{no} (*denied*), etc. O_{bc} indicates a word whose argument-requirement is the ordered pair B, C .

That N and O , without specifying subsets of them, suffice to characterize the argument-sets of all or almost all operators is supported by the fact that the set of argument-demands of all words is the set of all permutations of N and O from length zero to length 3 or 4. That is, there are words N with zero argument, words O_n with one N argument, O_{nn} with two (e.g. *ate* in *John ate fish*), O_{nnn} with three N (e.g. *put* in *John put the book on the table*, but not *John put the book*), perhaps O_{nnnn} (e.g. . . . *interpose . . . between . . . and . . .*), O_o with one operator as argument, O_{oo} with two operators as arguments (e.g. *entails*), O_{no} with N and an operator as ordered arguments (e.g. *denied*), O_{on} with the reverse order (e.g. *amused* in *John's phoning amused me*), O_{nno} (e.g. *told* in *John told Mary of Frank's phoning*), O_{noo} (e.g. *attribute* in *I attribute her leaving to his phoning*). These prove adequate as syntactic categories for the language.

Although the general conditions for operator entry are as given above, various languages may have a finite (and reasonably small) number of restrictions applying to particular subsets of operators. A subset of operators may be so expected in each sentence as to be virtually required (e.g. the *before*, *after* source of tense). In a particular subset, e.g. *continue*, *cease*, etc., the occurrence of one operator may preclude any other one—or itself iterated—from operating on it. However, strong as such restrictions may be in the overt grammar of affixes, etc., they generally turn out to be only selectional (sect. 5) and not rigid in the underlying word-entry operators.

Beginning merely with ordered word-entry to make a sentence, a fair amount of structure has thus been obtained. In summary: The entry of words into sentences depends upon the word's

argument-demand; the arguments are identified only by whether they themselves have arguments or do not have arguments, or are themselves zero, and not by belonging to particular subsets of these, or by other properties; and every resultant of an operator with arguments that satisfy its argument-demand is a discourse (and a sentence). From this it follows that the set of discourses is the set of all sequences satisfying the argument-demand relation. For the structural analysis, it is not necessary to identify the objects in the sequence as words, let alone words of a particular type, since their condition of entry depends entirely on their relation to objects defined in turn by their conditions of entry.

3. *Unrestrictedness*

We have seen (in sect. 2(2)) that, except for special cases (below), operators are not restricted to particular subsets of the N and O arguments. We will now see that we do not need to admit any subsets of operators distinguished by position in respect to their arguments. That is to say, given an operator set O_{bc} with argument-requirement BC , it will not be the case that one proper subset, O'_{bc} , makes sentences only by occurring in one position with respect to BC (e.g. $B O'_{bc} C$), while another, O''_{bc} , occurs only in another position (e.g. $BC O''_{bc}$). If this were the case, operators would have to be characterized not only by their argument-demand but also by their position in respect to their arguments, and one might think that each position indicated something different than what would be indicated by the operator words themselves. Many languages seem to have such position-subsets of operators; but it can be shown that there is one unrestrictive position in which there can occur all operators (or transformational equivalents of them in a manner indicated below) having a given argument-demand, and that any operators having that argument-demand which are restricted to other positions can be characterized as reductions (by section II) from particular operators in the unrestrictive position. The restriction becomes then a matter of subsets of operators receiving a reduction, rather than of subsets of operators having different relations to their arguments.

To show this requires a rather involved discussion. We begin with two properties of word-occurrence likelihoods: (1) For each operator word, some words in its argument-demand are more likely to occur as its arguments than are other words in the set; this likelihood-gradation differs from one operator to another, and is related to its

meaning, as in the more and less likely arguments of *slept*. Similarly for arguments, in respect to operators on them. (2) For certain O_{oo} (e.g. *cause*), what is most likely in the arguments is that their second argument or some argument further down under it, even if zeroed, should be the same word as their first argument or some argument under it in turn. Thus in any two sentences under *cause*, word-repetition is usually present or implicit (i.e. zeroed by sect. 8): In *The loss of his umbrella caused him to buy a new umbrella, umbrella* occurs in both operands of *caused* (i.e. in the two sentences operated on by *caused*). In *The threat of rain caused him to buy an umbrella* there is no repetition; but among the implicit sentences, including dictionary-definitions, which could have been joined to *rain* and have been zeroed are sentences such as *umbrellas protect from rain*. In contrast, *Its being Tuesday caused him to buy an umbrella* is unlikely, as containing no overt or zeroed repetition.⁶

We now raise the question of how it is that every language can express almost any information, though each has a limited vocabulary. To see how this capacity of language arises, consider the following: Let C_1 be a particular word, e.g. *house*, occurring as operator or as one of the arguments in all sentences of a set of sentences $A B C_1$. Some word pairs in the word sets A, B are most unlikely with C_1 ; the other pairs, which are not especially unlikely, will be marked $A_1 B_1$. Thus *John built, Fire burned* are in $A_1 B_1$ (yielding *John built a house, Fire burned a house*), but *John seated, Fire melted* are not (**John seated a house, *Fire melted a house*). To each sentence of the set $A B C_1$, we now adjoin $O_{oo} X Y Z$, where O_{oo} is an operator of the type in (2) above and $X Y Z$ is a particular individual sentence which is second argument of that O_{oo} . In the resultant set of sentences $A B C_1 O_{oo} X Y Z$ (where all symbols except A, B indicate particular words), the members of A, B which are not especially unlikely are not the same as those indicated by $A_1 B_1$. Thus, let $O_{oo} X Y Z$ be (a) *which is made of snow* where O_{oo} = semicolon plus *wh-* (relative

clause) and $X Y Z$ = *house is made of snow*, with *-ich* as pronoun for *house*; or, with reductions, (b) *made of snow*. The more likely A, B pairs here, marked $A_1 B_1$, include *John built, Fire melted* (as in *John built a house made of snow, Fire melted a house built of snow*), while other pairs such as *John seated, Fire burned* are not in $A_1 B_1$ (**John seated a house made of snow, *Fire burned a house made of snow*). The difference between $A_1 B_1$ and $A_j B_j$ arises because under the O_{oo} there exists greater likelihood for those $A B$ words which are semantically related to the fixed $X Y Z$ words. Thus the particular word-sequence $C_1 O_{oo} X Y Z$ determines a different likelihood-gradation in $A B$ than does C_1 alone: $A_1 B_1$ instead of $A_j B_j$. Indeed, by (1) above, $C_1 O_{oo} X Y Z$ acts like a new member occupying the C position in $A B C$. In this way, grammar creates word-sequences that do the work of an indefinitely expandable vocabulary. Today, English vocabulary contains a word *igloo*, borrowed from Eskimo, which determines approximately the same operators on it (and co-arguments under those operators) as does *house made of snow*. But the possibility of having the likelihood-gradation which is characteristic for operators on *igloo* (and so the meaning of that word) did not require the existence of that word in English; it was available for *house made of snow*.

To return now to the restrictive positions of operators. Most operators O_o on a sentence, hence on an operator, occur after their argument, e.g. *continued, is an art*, in *John's solving puzzles continued, John's solving puzzles is an art*; these will be indicated here by O'_o . Only a few, O'_o , occur at an interior point of their operand, between their immediate argument and its argument in turn: e.g. the auxiliaries *can, may*, etc., and also such verbs as *continued*, in *John can solve puzzles, John may solve puzzles, John continued solving puzzles*. Following the discussion above, for every O'_o with its particular likelihood-gradation on its operand sentences, it is always possible to find a paraphrasing $O'_o O_{oo} X Y Z$ such that the likelihood-gradation which it imposes on its operand sentences differs from the

⁶ Zellig Harris, *Mathematical Structures of Language*, Interscience Tracts in Mathematics 21 (New York, 1968), pp. 131-135. The finding is that in every occurrence of *cause* between two sentences which is considered likely (or, in terms of hearer's response, acceptable) either such repetition is seen, or there are zeroable (implicit) sentences, such as definitions or other likely sentences, adjoined to one of the arguments, which supply the repetition. Such a sentence would be *The threat of rain, for which umbrellas are a protection, caused him to buy an umbrella*. Definitional sentences joined to an argument by semicolon plus *wh-* (relative clause) are zeroable (*ibid.*, pp. 78-83, 137-138). *Its being Tuesday caused him to buy an umbrella* is felt as there being no semantic connection which would explain the casual relation; but grammatically what we see is the absence of overt or grammatically-zeroable repetition. One could explain the causal relation by adjoining to *Tuesday* the sentence *and rain was predicted for Tuesday*; but this would not be zeroable by the reductions of section II below. Had the sentence been *Its being Tuesday, with rain predicted for Tuesday, caused him to buy an umbrella* (with zeroed *which protects against rain* at the end), the sentence would seem perfectly likely, with zeroed repetition of *rain*.

likelihood-gradation imposed by O'_o and approaches that of the given O'_o as closely as we wish. The only question that might arise is if the O'_o have a restrictive grammatical property (aside from their individual likelihood-gradations) which does not hold for the O''_o . Indeed, the O'_o carry a special reference to the first argument of their operand sentence: *continued* refers to *John* more directly in *John continued to swim* than in *John's swimming continued*. However, the fact that we can adjoin to a sentence metalinguistic references to any part of that sentence (sects. 4 and 16) assures the possibility of adjoining to the selected O''_o O_{oo} $X Y Z$ a metalinguistic statement giving it the O'_o property. Thus for *continued* in O'_o we may say *was continued by the subject*; for *can* in O'_o we may say *is a capability of the subject's* (*John can swim, John's swimming is a capability of the subject's*). In contrast, we cannot construct in the more restrictive O'_o position a paraphrase of each O''_o , because no metalinguistic statement on O'_o words can remove from them the special property (the subject reference) which adheres to being in the O'_o position.

It follows that for each argument-demand, there is a position, which in English is found to be always after the first argument, in which every operator having that argument-requirement either itself occurs or else has a paraphrase which is equivalent to it in likelihood-gradation and metalinguistic properties. It will be seen below that in English the equivalents in the post-first-argument position satisfy the conditions for being the base form in respect to which the restricted operators are reductions (transformations). Thus we can say that the post-first-argument position is unrestricted, containing all operators; and that for some operators there take place reductions (e.g. to *can*) which send them to another position, the reductions being restricted to particular members of the operator set.

We have thus arrived at a system of operators which are not restricted, in respect either to subsets of their arguments or to the position of the operators. There may nevertheless remain, in a particular language, certain subsets of operators which are restricted. Thus, for a particular set of operators which are restricted in their position (such as the English auxiliaries), or in their ability to occur under certain further operators (such as the aspectually-selective verb stems in Slavic languages), it may be difficult to find satisfactory unrestricted paraphrases; or these paraphrases may be too complex or metalinguistic. Also, some sets of operators may be such that even in their most unrestricted

paraphrase they cannot iterate with themselves or with each other. In all these cases, we are dealing with specifiable subsets of operators whose members have (approximately) identical or complementary inequalities of likelihood of occurrence in respect to another specifiable subset of operators on them (e.g. certain operators describing number, durativity); and the members of the subsets would have to be listed.

4. Metatextual Operators

There are a few operators or operator-argument combinations which differ from all others in that they name the entry-order of a word in the sentence on which these operators are acting: that is, they refer to a word within their own argument, and will be termed "metatextual" (or "metadiscourse"). Such referring clearly exists in language, e.g. *He likes Mozart and Bach, but I prefer the latter*, where *latter* refers to the position of the word *Bach*. The possibility for such referring is inherent in the ordering of word entry, mapped (though with various changes, section II) onto the linear order of words in a sentence. The presence of metatextual operators may be seen if we consider, say, *A glass tipped and a glass fell*. If we want to give the information that the two glasses are the same, it is possible to use the inherently available address: *tipped* and *fell* are the first and second arguments, respectively, of *and*; *glass* is the (first) argument of each of these. Then under *and* we address *tipped* as 1, *fell* as 2, the first *glass* is 1.1 and the second is 2.1. The information about sameness here would be given by having on *and* an operator *has the same referent* (or *indicates the same individual*) whose arguments are the contents of the addresses 2.1 and 1.1. With this sameness-operator we obtain reductions to *A glass tipped, and it fell* or to *A glass tipped and fell*. These operators may seem to be peculiarly complex and merely a figment of theory, but they are inescapable for a simple analysis of language. For, as will be seen in section II, such operators yield in a natural way the phenomena of zeroing and pronouncing, without appealing to any grammatical apparatus beyond what exists otherwise in grammar. Without these operators, one would have to announce the fact and conditions of zeroing and pronouncing in various grammar-statements, i.e. statements made in the grammar about the sentences of the language. However, if we consider such grammar-statements (which are themselves English sentences), then we see that their information can be given by metatextual operators of English attached to the very

sentences which the statements are describing. Naturally, the information about sameness cannot be given until the two words which are the same have both entered the sentence: hence we need an O_o operator (*has . . . same . . .*) on the O_{oo} (*and*) which had brought the two words together in the sentence. This O_o sameness-operator can also act on O_{nn} (e.g. on *John washed John*) in which case it leads not to zero but to *self*-pronouns: *John washed himself*. And when the sameness-operator specifies which words are the same, the simplest way of addressing them is by their entry-order in the operator history of the sentence; the word-order in the sentence derives from this, but in a complex way.

Particularly important are the *wh*-pronouns, which create the relative clause, and all those sentence-segments that the grammarians would call modifiers—whether on noun, or verb, or sentence, or whatever. They also underlie all cross-reference. Operators on a sentence can become parts of a “noun-phrase” or any similar “word-phrase” (i.e. a word with its modifiers) only via these. Syntactically, these *wh*-words are the form that pronouns can take when they follow semicolon, which is an O_{oo} connecting two sentences: *The man who had been here left* from semicolon on a sentence pair: ← *The man left*; *The man had been here*, with pronouncing of the second *man* into *who*.⁷ The pronouncing, of course, arises from a sameness-operator on the two occurrences of *man*: *The man left*; *the man had been here*; 2.1 indicates the same individual as 1.1. Similarly, *The man whom I saw left* is a reduction of *The man left*; *I saw the same-as-1.1 man*. ← *The man left*; *I saw the man*; 2.2 indicates the same individual as 1.1.

Introducing a sameness-operator as a base for zeroing and pronouncing makes it possible to obtain these as reductions from a metatextual operator inside the base sentence (section III) with no apparatus of operators and reductions beyond what is in any case needed to obtain the referential effect of words such as *latter*. Nevertheless, it makes the base sentences look artificial. Instead of this, one could therefore take pronouns not as metatextual changes (*it* for *glass*), but simply as words of

referential or deictic meaning: *he* meaning “a known or mentioned male person,” etc. And instead of some, but not all, zeroing we could have *and* acting on operators or arguments before these meet (*tipped and fell* acting together on *glass*). Such analyses would make the base closer to the usual language, but would reduce the simplicity of the system that can produce all sentences.

5. Likelihood-Inequalities

The crucial relation in language structure, after the sentence-making ordered entry, is the fact that every operator-word has a particular likelihood-gradation, more precisely inequalities of likelihood, for the words in each of its argument positions, and vice versa. Since we can hardly measure the likelihoods of, say, each *N*'s occurring under *slept*, and so on, we have to deal with the estimate of likelihood as made by speakers of the language, i.e. with the presumed frequency of occurrence. Even so, no precise data can be obtained for all *N* in a particular position, but it is sufficient to work with several grades such as especially low (vanishingly small) likelihood (e.g., under *slept*, for *vacuum*), rather low likelihood (e.g., for *ocean*, *rock* as in *The oceans slept*), somewhat-lower-than-ordinary likelihood (*The flower slept*), ordinary likelihood (*The boy slept*); the last (ordinary) is often called the selection for the given verb in the given position. In addition, some operators have words which are exceptionally likely to occur as their arguments, e.g., *to be here* as argument of *expect* (sect. 7). One may think of intermediate grades, as for *The tree slept* (just below ordinary likelihood?), and one may be uncertain in some cases (e.g., does *earth* have ordinary likelihood under *slept*, as in *The earth slept under the blanket of snow*). The gradation is, of course, related to meaning, but by no means precisely, not only because of meaning extension and explicit metaphors, but also because of the special frequency of certain literary expressions (e.g., *The earth slept*), and so on. While the likelihoods themselves change readily in time and differ as among speakers, the gross classification is more stable. Above all, the inequalities in likelihood are preserved under transformations (below).

⁷ The alternative would be to say that the segment headed by *wh*- is something new, to be called a modifier, which is added directly to a noun: as though *The man who was here left* is formed from *left* operating on *The man who was here*, and *The man whom I saw left* is formed from *left* operating on *The man whom I saw* (these noun-phrases being formed in turn from *who was here*, *whom I saw* being added to the *man*). But if we consider all possible *wh*-modifiers on a particular noun N_1 we find that they are simply all sentences containing N_1 , with N_1 omitted: e.g. *was here*, *I saw*. There is no independent structural characterization of what segments can be added to N_1 ; we have to say that these segments are sentences which contain N_1 and in which the N_1 has received zero (or *-om*, etc.) shape.

Instead of speaking of inequalities of likelihood for the arguments of an operator, one can speak of what further operators on the given operator (which, as in sect. 7, includes what context of neighboring sentences, etc.) would make a particular word not low in likelihood. For not otherwise specified likelihoods, this context is ordinary speech and writing. For specialized word uses, e.g., in science, this context is the neighboring material of scientific articles or conversations. For certain kinds of rare word uses, such as in *The saucer laughed*, the context can be neighboring fairy-tale sentences. And for ungrammatical word sequences (i.e. ones not satisfying the argument-demand relation) the context is a metalinguistic operator such as *word* (as in *He said "Go the a,"* reduced from *He said the words "Go the a";* note that one cannot say \exists *He said that go the a*, which is not reduced from a zeroed operator *word*, and in which *go the a* would be therefore occurring not under a metalinguistic operator).

Two items of background should perhaps be given about this appeal to likelihood:

(1) Unstructured attempts to characterize the likelihood of word-combinations have failed when they related merely to neighboring words in a sentence, or the like. But when we deal with relative likelihood (i.e. inequalities of likelihood) of arguments in respect to their operator, or vice versa, the likelihood differences begin to fit closely with meaning differences, and to be preserved under sentence transformation.

(2) Likelihood inequalities of operator-argument combinations do an important part of the work of indicating the grammaticality of sentences, i.e. approximately the work of characterizing which sentences are in the language. Students of language have avoided considerations of frequency and likelihood, because these were thought to be unrelated to structure. Thus, there are some sentences which one is certain were never said, but which are immediately understood as grammatical: e.g. *Some blue and mauve onion-skin shot through the air at 759.06 miles per second.* However, such unlikely yet fully grammatical sentences have a particular relation to likely sentences: likely sentences can be formed immediately from these by substituting classifiers for the specific words and adding sentences stating that the specific words are cases of those classifiers. Thus we can form: *Some colored*

solid object shot through the air at a particular velocity; Blue and mauve are colors; Onion-skin is a solid object; 759.06 miles per second is a particular velocity.

There are other kinds of sentences whose likelihood seems unrelated to their grammaticality. Here too, it is possible to find that the different ways and degrees of a given sentence's grammaticality correspond to different sentences of non-low likelihood to which the given sentence is syntactically related. Thus new or far-fetched metaphors, jocular and nonce uses of words, and marginal sentences whose grammaticality is uncertain, can all be characterized as particular kinds of combination of, or departure from, corresponding sentences of ordinary non-low likelihood.⁸

The likelihood-inequalities of arguments for each operator, and vice-versa, differentiate the operator words and their meanings. Some operators have special properties in this regard, by virtue of which they can be collected into subsets in respect to selection. Thus, some have very broad selection, i.e. most or very many of the words in their argument-demand have non-low likelihood of occurring under them: e.g. under *before*, *after* more different words are likely to occur than under *to the right of*, *on top of*. Some have each a favored ("appropriate") argument which is the most likely one under the given operator, e.g. *to be here* under *expect* (sect. 7). For some, what is most likely is not that the argument should contain particular words rather than others, but that there be some word which occurs in each of their arguments, at whatever depth: so for the O_{oo} in sect. 3. There are also cases of various operators having similar likelihood-gradations. Thus the selections of *before* and *after* are very similar though not identical; similarly as between *is more than*, *is less than*, *equals*. Certain O_{oo} (chiefly *and*, *or*) have in common the property of being often semantically commutative and associative; their non-commutative and non-associative occurrences can be obtained from zeroing such words as *then* (sect. 7). That is to say, *He is tired and he is hungry* means much the same as *He is hungry and he is tired*; and if the arguments of *and* are *He is hungry* and *he is tired*, *He is sleepy* the resultant sentence and meaning are the same as if the arguments are *He is hungry*, *He is tired and he is sleepy*: both yield non-ambiguous *He is hungry and he is tired and he is sleepy*.

There are subsets of operators some of whose

⁸ Zellig Harris, *Notes du cours de syntaxe*, ed. by Maurice Gross (Paris, 1976); and *Transformational system of English: information and reduction*, in Formal Linguistics Series, Reidel, Dordrecht, to appear. These volumes contain data needed to support the present theory as a grammatical analysis.

members have similar selections, while others have contrasting selections. Thus a number of O_0 operators have in common the following properties: they have a broad selection; and the likelihood-gradations of their arguments are much the same under all of them. Other O_0 operators have a likelihood-gradation for arguments which is almost the reverse of that for the set above. Thus under *lasted*, *continued*, we find that *eat* has ordinary likelihood and *arrive* low likelihood: *His eating continued*, but hardly *His arriving continued*; but under *occurred*, *was sudden* the likelihoods are reversed: *His arriving was sudden*, but hardly *His eating was sudden*.

Certain operators, *is a word*, *is a sentence*, *is an O_{no} operator*, etc., are uniquely distinguishable in language analysis, and form the base for the metalanguage, even though they may have in the language itself a status not different from that of other classifiers such as *is a mammal*, *is an atom*. This is because their argument selection is approximately the set of all words, all sentences, all O_{no} words, etc., respectively. However, the arguments are precisely determined only in the field of language-analysis, where we consider only the arguments which together with *is a word* or the like make a scientifically reasonable sentence (*Book is a word*, but not *She spoke is a word*). In the language itself one can find *It is*, *in two words*, *impossible*, and other non-word arguments of *is a word*, just as one can find *A whale is a big fish*. These classifiers, even the metalinguistic ones, do not form clear-cut subsets of operators in the ordinary language.

These various argument-likelihood properties give a texture of similarities to the operators on a given argument-demand. Except as noted at the end of sect. 4, they fall short of creating subsets of operators, because for many of the properties there are operators which have the given property to a lesser degree or in a different manner than the other operators. However, they have the effect of creating semantically important, though fuzzy, categories of the operands: e.g. durative verbs (*eat*) as against momentaneous (*arrive*). And these properties are structurally important in being the basis for the reductions (section II).

It must be stressed that the likelihoods are not used directly in constructing a grammar. The major use is that preserving the inequalities of likelihood—in a specified way—is a proved criterion in determining the transformations of a language (III). For the rest, the discussion of likelihoods has an interpretational status, in characterizing the transformationally-established reductions.

II. REDUCTION

The importance of the sentence-making operations described above—the word-entries satisfying their argument-demands, the argumenthood-indicators, and the likelihood-inequalities—is that all remaining sentences of the language can be obtained from these by a few physical types of reduction in the shape, or relative position (relative distance), of the words which enter a sentence (fn. 8). The great bulk of reductions can be made on an operator, or its argument, if one of these has exceptionally broad selection or exceptionally high likelihood of occurrence in respect to the other, or if it otherwise contributes little or no information at its point of entry in the construction of the sentence. The reductions are thus not arbitrary changes. Most or all have a common property, based on the relative likelihood property of the operator-argument relations, of reducing high-likelihood low-information entries. It is also found that all, or most, of the changes (which will be seen below to constitute sentence transformations) can be best obtained if we assume that the reductions are carried out as the operator enters the sentence or as the specified conditions for the reduction come to be satisfied by the sentence as so far constructed. This greatly simplifies any computational or effective procedure for sentence transformations. It means that if a reduction has taken place on a given operator, the next operator to enter the sentence enters upon the reduced form of the earlier operator.

Not all low-information entries occasion a reduction. These have to be discovered in each language, although some types are widespread. There are three main types of reduction: zeroing and pronouncing for operators or arguments which have certainty or highest likelihood of occurring at a particular point in the construction of the sentence (sects. 6–8); affixation and intonation in the case of operators which have broad selection in the language (sect. 9); moving of metatextual material to the address to which it refers (sect. 10). All reductions, including zeroing, leave a trace which is seen in the entries immediately environing the one which was reduced. Where there is no such trace, no reduction is defined.

6. *Pronouncing and Zeroing for Repetition in the Discourse*

The metatextual O_0 which assert sameness of two addressed arguments (sect. 4) make one of the

occurrences of the repeated word certain, given the other occurrence, as in the two occurrences of *glass* in sect. 4. The identity of the word at the given location can therefore be dispensed with, as being recoverable from the assertion of sameness on two addresses. In some situations, especially when the other address is unrestricted (i.e. when the other occurrence could be anywhere under the meta-textual operator), the repeated word can receive a shape which is tangible but which is not individual to that word: a pronoun. E.g. for *John's winning was due to John's endless training* (plus sameness-operator) we have both *John's winning was due to his endless training* and *His winning was due to John's endless training*.⁹ In other cases, especially when there is a fixed entry-relation, hence fixed initial distance, between the two occurrences of a word, the repeated word can receive zero shape.¹⁰ E.g. from *A glass tipped and a glass fell* (plus sameness operator) we have *A glass tipped and fell*; from *He will buy a book if she will buy a book* (where the sameness-operator is stated about the entering words, not the individual referents) we have *He will buy a book if she will*. Although the occurrence of the repeated word is certain, given the sameness-addresses, different sentences with different sameness-addresses may yield the same pronoun locations, or the same word sequences after zeroing. Hence ambiguities arise among sentences, i.e. degeneracies of word-sequence in respect to the ordered word entries: e.g. *I left him feeling sad* from *I left him, I feeling sad*, and also from *I left him, he feeling sad*.

7. Zeroing for Highest Sentence-Entry Likelihood

When an operator enters upon its arguments to make a sentence, the argument can be zeroed if it is by far the likeliest (the "appropriate") argument for the given operator; or the operator may be zeroed as most likely or appropriate on its arguments.¹¹

A simple example is the argument of *expect*. The second argument of *expect* is generally an operator

(equivalently, a sentence): *We expect their departure at 6*, *We expect them to depart at 6*. However, we also find certain elementary arguments as second arguments: *We expect John*. The inequalities of likelihood for *N* as second argument of *expect* are approximately those for *N* as argument of *is here* or the like: *John is here*, **Time is here*, **We expect time*. Also, if for each sentence having *N* as second argument of *expect* we form a corresponding sentence having *N to be here* as second argument of *expect*, we find the two corresponding sentences to have approximately the same inequalities of likelihood for various further operators on them, i.e. for various contexts. The two inequality-similarities above are precisely the conditions for one form to be a transform of the other (section III), and we can consider *to be here* to have been zeroed under *expect*, as being the most likely argument of *expect*.

Similarly, under the operator *say*, whose last argument-position can be taken by any operator (i.e. any sentence) but also by metalinguistic classifiers such as *word*, the latter have a uniquely appropriate likelihood and are zeroable. Thus *wh-operating* on the pair *John said words*, *It is late are words* produces *John said the words it is late*, zeroed to *John said "It is late"*; here *words* was the second argument of *said*. But when *It is late* is directly the second operand of *said*, we have *John said it was late*, *John said that it was late*.

A simple example is the zeroing of *then* after *and* if the two arguments of *and* are clearly time-ordered: *He took sick and died* ← *He took sick and then died*. Many complex structures in the grammar are obtained with little or no further ado by the zeroing of especially likely, appropriate, arguments. Examples are the zeroing of *amount*, *degree*, and the like, under quantifier and comparative operators such as *increase*, *is more than*, *is less than*. So also the zeroing of *moment*, *period*, and the like, under time-order operators such as *before*, *after*; this simplifies the grammar of tenses and aspects.

Somewhat differently, an indefinite appropriate

⁹ There is also a special pronoun from the *Speaker says* operator: *I, you* are repetitions of the two *N*, respectively, in the *N₁ says to N₂* which can operate on any discourse or part of it (below): *N₁ says to N₂: N₁ wanted to see N₂ → N₁ says to N₂: I wanted to see you*; with zeroing of *N₁ says to N₂* (section 8, below) we obtain *I wanted to see you*.

¹⁰ This zeroing is almost always on the second occurrence, e.g. in parallel positions under the commutative *O_{oo}* (*and*, *or*). Repetitional zeroing occurs in specific positions of specific argument-demands e.g. also under *O_{no}* when the first argument of the operator which has become an argument is the same as the first argument of the operator on it: for *John prefers for John to stay in* (under a sameness operator) we have *John prefers to stay in*.

¹¹ In many cases, an operator is zeroed only after a further operator has acted on it. The zeroing of an operator when it is the last operator to act would in many cases leave no trace of its having been present; we might then have no evidence that it had been zeroed. The occurrence of zeroing can also be restricted by demands on the word-sequence. Thus in English, indefinite second arguments are zeroable, but not (in general) indefinite first arguments: *I read* from *I read things*, but not *∃ Eat too much* from *People eat too much*.

second argument can be zeroed under many operators: *I read* is a reduction from *I read things* or the like.¹²

8. Zeroing for Assured Occurrence or No Information

Certain operators, together with a part but not the whole of their operand, are zeroable when they are the only words that could be occurring in the given entry position. This applies to relative-clause *wh*- words plus *is*, as in *The man here has been waiting* ← *The man who is here has been waiting*; and the same with the indefinite *that*, *anything*, or *group* or the like, preceding such *wh*-words, as in *I never eat her cooking* ← *I never eat anything which is her cooking*. This accounts for the difference between *Mozart and Beethoven wrote operas*, obtained by zeroing from *Mozart wrote operas and Beethoven wrote operas*, and *Gilbert and Sullivan wrote operettas* which must have a different source, namely *The team which was* (or: *contained*) *Gilbert and Sullivan wrote operettas*, from *wh*-operating on *A team wrote operettas*, *The team contained Gilbert and the team contained Sullivan*.

When *will*, *-ed*, are introduced onto the first arguments of *after*, *before*, and period intonation onto the first argument of *and*, *for* (sect. 9), then the original portion of the operators, namely *after*, *before*, *and*, *for* can be zeroed: *He did it before* is reduced to *He did it*. Similarly *He was sleepy. And he was hungry* is reduced to *He was sleepy. He was hungry*.

The *I say, I report* (or N_1 *says*, where N_1 is the speaker) which can be assumed on every discourse, can for that reason be zeroed. Many difficult forms are explained by this. One can also assume that discourses carry adjoined sentences or operators which give all the situational and definitional information necessary to the understanding of the discourse.¹³ These are zeroable because they are already known to the hearer, and thus their having been present (as necessary explainers of the sentence) is a certainty; the trace of each such defini-

tion is the presence (in the sentence) of the defined word. Among the many simplifications that this assumption brings to grammar is the fact that deictic pronouns (e.g. in *This is expensive*, with *this* referring to something known only from the non-verbal situation) become under this assumption simply cross-referential (repetitional) pronouns: e.g. we can begin with something like *We are considering an object here. The object is expensive* (under the sameness operator), reduced to *We are considering an object here. This is expensive*, to merely *This is expensive*.

Also zeroable are operators (some with their first arguments) which are informationless in given situations, where they are similar to the performatives discussed in the philosophy of language. It has been shown that *He asks whether it is late* is reducible to *He asks: Is it late?*, and *I requested you that you please go* to *I requested you: Please go!* (sect. 9). The *asks*, *requests* occur with various first arguments and tenses: *You asked*, etc., and are not zeroable. However, *I ask*, *I request* are unique in bringing no information into the sentence, and are zeroable. For, saying *I ask you: Is it late?* constitutes asking the question and thus says the same thing as *Is it late?* by itself. Similarly, *I request you: Please go!* constitutes making the request and is thus the same thing as saying *Please go!* by itself. Much in the grammar of questions and imperatives (and wishes, etc.) is simplified when they are taken as reduced from *ask*, *request*, etc.; and the zeroing of these operators is reserved for the case with the informational properties noted here.

9. Broad Selection in the Language

Certain non-elementary operators, which have a good likelihood on almost all operators of the language as their arguments, can attach affixes or intonations to their arguments; as seen in sect. 8, the original operators can then be zeroed. And certain operators which have a broad selection,

¹² The contexts of *I read*, i.e. the further operators on it, show that it means not all reading activities (as in *I read everything*) but reading whatever it is that one would be likely to read. Some operators do not have this second-argument zeroing, and these seem to words which are not likely to be said with an indefinite appropriate object: e.g. we don't say *I wear*, nor is one likely to say *I wear things*.

¹³ As an indication of this, consider the following: A grammar can be looked upon as a device which decides which sequences of phonemes, or of words, is a sentence or discourse of the language, and which is not. But we can take an arbitrary sequence of phonemes (satisfying the phonemic structure of the language) and add to it metalinguistic operators which say that the first few phonemes are a person's name, the next few phonemes (say, ending in a phoneme that can be a tense suffix) are a specialized biochemical term (verb) meaning to carry out some particular laboratory operation, and the remaining phonemes are the name of some new chemical compound. Then the phoneme sequence is a sentence of English. When phoneme sequences are accepted as sentences of English without such explanations, it is because these explanations are known to the hearer. We can assume that they existed as operators on the sentence, no less than in the case above, but were zeroed because they were known, i.e. their presence as metalinguistic operators on the given sentence was assured.

though not as broad as the set above, are reduced to being affixes on their arguments; these affix forms in most cases have no phonemic similarity to the operator, and must thus be justified as suppletive transforms. In detail:

The time-order words, chiefly, *before*, *after*, can impose *-ed*, *will*, respectively on their first argument, *A*, replacing the operator-indicator *-s* on *A*. When the *-s* has not been replaced, *A* is understood as "present" or "timeless". An efficient explanation of both the temporal and the non-temporal uses of the tenses (including their aspectual properties and such special uses as the past for expressing contrary-to-fact) is obtained by deriving the tenses not from subjective time, but from the time-order of an operator to another operator in the sentence. As a rough example, consider (1) *He will have arrived before your return*. For demonstration, we take the *will* as already present, and account first for the *have -ed*. The source would be semicolon plus *wh-* operating on (2) *He will arrive* and (3) *His arriving is before your return* (where *is* is not present tense, but timeless). This produces *He will arrive, which is before your return*. Here the *which is before* permits a past-tense marker to be added to its host *arrive*; and the *which is* is zeroable (sect. 8), yielding (1). As to the tense on the latest-entering operator, here *arrive*, that results similarly from its time-order to the *I say*, *I report* which can operate on all sentences or texts, and which is later zeroed. We begin with semicolon plus *wh-* on *I report his arriving* and *His arriving is after* (or: *subsequent to*) *my reporting*, producing *I report his arriving which is after my reporting*. On this there operates a metatextual operator stating that the second *report* refers to the first: *I report his arriving which is after my reporting*; 2.2 has the same referent as 1.1. This permits zeroing of *my reporting*; together with zeroing of *which is*, this yields *I report his arriving after* (or: *subsequently*). Independently, the *which is after* permits a future-tense marker on the host *arrive*. With the coming of the tense marker, the argumenthood-indicator changes from *-ing* to *that*, so that *his arriving after* changes to *that he will arrive after*. When the tense-marker is in, the *after* is zeroable; and *I report that* is zeroable, yielding (2) *He will arrive*.¹⁴

Tortuous as this derivation is, it shows that even the tenses express nothing that cannot be expressed by the operators defined in this theory. The tenses satisfy the conditions for being reductions

(transformations) of *before*, *after*, etc., in particular situations.

The operators *ask*, *request* can impose on the sentence which becomes their operand an intonation of question (permuting tense and first-argument word) and command respectively: *He asks whether it is late*. → *He asks: Is it late?* (sect. 8).

Except as above, discourses end in period intonation. In addition, a number of *O₀₀*, chiefly *and*, *for*, but also *or*, *but*, *because*, *if*, etc., can impose a period intonation on their first argument: *John refused for it was too late*. → *John refused. For it was too late*. Most reductions can then not occur across the period; but pronouncing and certain other reductions can. This is sufficient to segment the discourses into sentences. Then the zeroing of *and*, *for*, by sect. 8, yields *John refused. It was too late*.

Certain broad-selection operators, chiefly ones having the form of preposition or the negative, which enter before an argument or are permuted to that position, can be reduced (usually suppletively) to being prefixes on their argument: *It is under normal* → *It is subnormal*; *It is a half circle* → *It is a semicircle*; *It is less than finished* → *It is unfinished*.

Certain broad-selection operators, meaning *condition*, *state*, *tendency* and the like, take in English a position after their argument (in many cases by the "compound-noun" permutation of sect. 10) and then reduce to suffixes on their argument: *His childhood was happy*, via a non-used **His child-state was happy*, from *His state of being a child was happy*, from *happy* operating on *His being a child was a state*. The argumenthood-indicator makes this into *The state of his being a child*, *His state of being a child*, as it makes *His being absent is a trick* into *The trick of his being absent*.

It appears that the affixes of English can be related in this way as suppletive (i.e. phonemically dissimilar) transforms of operators on the words to which the affixes are then attached. In some cases (most prefixes, and some suffixes including the *-hood* above), the affixes are historically not suppletive but reduced shapes of those operators (fn. 8).

10. Permutation to Referred Address

Given that each operator enters into the fixed position after its first argument, most permutations in English are cases of a metatextual operator or second argument, which contains an address (sect. 4), moving to the address to which it refers. Thus,

¹⁴ This analysis provides the same base for tense-consecution and other exceptional uses of tenses as for the ordinary tense on the main verb of a sentence.

a semicolon sentence (especially if it contains a *wh*-pronoun) moves to after its referred address: *My friend returned; I had mentioned him to you* → *My friend—I had mentioned him to you—returned*. Also *My friend returned, whom I had mentioned to you* → *My friend whom I had mentioned to you returned*. When something is zeroed in the second argument of *and*, the residue (i.e. *and* with what is left of its second argument) moves to after the last non-referred material in the first argument: *John saw Mary and John phoned Mary* (under the sameness- O_0) → *John saw and phoned Mary*. After zeroing of *which is*, and *who is*, the residue, if it consists of certain sets of short words such as adjectives, moves to before its referred address: *The pen which is blue writes well* → *The blue pen writes well*. In English, if the residue is of the form of a preposition (chiefly *of*) plus noun (including words made noun-like by their affixes), then the preposition is dropped and the residue moved with compound-noun stress to before its referred address: e.g. this occurs twice in *The machinery is for the building of roads* → *The machinery is for road-building*, and *The machinery for road-building failed to work* → *The road-building-machinery failed to work*. The first compound-noun form here, *road-building* is part of an argumenthood-indicator; the second, *-machinery*, is from *wh*: *machinery which is for someone's road-building*.

There are a number of O_0 operators which appear not after their argument but before their own argument-word and after that word's argument in turn: *He stopped running* (as against *His running stopped*); the auxiliaries, as in *He can run*; the negative in *He is not Greek* (compare *His being Greek is not so*, or the like). In some cases there are grounds for saying that O_0 after its argument carried a referent to the subject of that argument, and so moved to it: *His running stopped due to the subject*, or the like, → *He stopped running* (sect. 3). But in other cases, such as the *not*, such an explanation for the permutation is not available.

11. Reductions Unrelated to Amount of Information

There seem to be some reductions, perhaps only permutations, which have no basis in low informa-

tion. Such is the moving of certain short sentence-segments to before long ones: We have *I saw people near him*, but not **I saw near him people*. However, when long modifiers (i.e. *wh*-residues) are attached to *people* we find both *I saw several uninvited people near him* and *I saw near him several uninvited people*. There are also special permutations for certain adverbial words: *So they say, Hardly had they come than he left*.¹⁵

III. THE ENTRY-AND-REDUCTION SYSTEM

The essential fact is that the reductions (II), when applied to the ordered-entry discourses (I), suffice to characterize effectively all the sentences of the language. Since the entries are ordered, and the reductions of a word take place at its entry or at the entry of the immediate operator on it, or are referred to these, we have a decision procedure for constructing and analyzing each sentence. For construction: we have particular partially (but mostly linearly) ordered entries satisfying argument-requirement, and reductions satisfying their specified low-information conditions. For analysis: since zeroing is not a loss of a word but simply a zero shape for a recoverable word, each sentence can be directly transformed and segmented into partially ordered reductions and entries.¹⁶ The degeneracies in some pronouns, zeroings, and other reductions, in which different reductions on different operators yield identical word sequences, cause ambiguities; this only means having more than one analysis for the given word-sequence, although determining the various analyses may be difficult. Aside from the local cases of unordering among certain semicolon entries, the entries can be ordered with their arguments in parentheses. And since every word has stated requirements (one or more) as to its argument-sequence, the entry order of words in a particular sentence can be represented in Polish notation.

From the definitions of the argument-demand sets of words, it follows that every sentence must contain at least one elementary argument N , which alone can enter without prior entries; and for

¹⁵ The vocabulary also contains certain variants of word-form, called morphophonemic variants, which affect the shape of a word or affix under particular operators or on particular arguments, and which are not reductions and are unrelated to amount of information: e.g. *knife, knives*. The obligatory transformations (which are few, if any) and the morphophonemics are presented in the base sentences as sets of variant forms which certain words take under stated entry conditions.

¹⁶ The partial ordering arises, in the case of reductions, from independent reductions on the same entry. Algorithms capable of analyzing the structure of virtually all sentences of English in the sense of the system presented here have been written. Some have been successfully implemented as computer programs which carry out sentence analysis. Complexities in stating the domain of certain reductions, and degeneracies, make sentence-analysis no simple matter.

every n co-entering elementary arguments it must contain one n -argument elementary operator. Hence it contains at least one elementary sentence. And for every operator or operator-pair of whatever kind, it may contain a non-elementary operator on it.

Every operator makes a sentence. Every non-elementary operator acts on a sentence or sentence-pair (namely, on the sentences made by the operators which are the arguments of that non-elementary operator), and makes a further sentence. Every reduction acts upon a sentence, and results in an altered sentence. Thus all non-elementary operators and all reductions make sentences out of sentences, and are transformations in the set of sentences. The non-elementary operators are a set of transformations on the set of base sentences $\{S\}$ as constructed in section 1. Each non-elementary operator acts on all sentences (since there are few restrictions, but rather inequalities of likelihood, on the sentences in its argument positions), mapping the whole set of sentences $\{S\}$ into $\{S'\}$ (onto a subset having that non-elementary operator as last entry), preserving the inequalities. Each O_{oo} maps $\{S'\} \times \{S'\}$ into $\{S'\}$. The non-elementary operators act also on reduced sentences, not only on the unreduced ones of section I. But in this case there are restrictions, e.g. there is no *and* on the pair: question, assertion; hence here we have only partial transformations. The reductions are a set of partial transformations on the full set of sentences $\{S'\}$, each mapping a subset of sentences (those containing a particular low-information entry) onto another subset (those containing the reduction), preserving the inequalities. The way inequalities of likelihood are preserved under non-elementary operators allows for a limited amount of exceptions: as an extreme example, the operator *not* changes the relative likelihoods of various kinds of sentence (e.g. of general sentences) but preserves the inequalities of the great bulk of ordinary sentences (*John will leave, John will not leave*, both normal, as against *Vacuum will sleep, Vacuum will not sleep*). In contrast, the preservation of inequalities of likelihood under reductions is much stronger, leaving room for few word-choice exceptions.

Preserving inequalities of likelihood of operators

in respect to arguments guarantees preserving meaning in each sentence, aside from a reasonably small number of exceptions (idioms and the like). The non-elementary operators preserve the meaning of the sentence on which they act, and add their own meaning in respect to it. The reductions preserve the meaning and add no objective information. They are thus more or less paraphrastic transformations. In addition, any individual sentence may have various non-transformational paraphrases based on synonyms and circumlocutions special to its words in their neighborhoods. These paraphrases do not preserve inequalities, and do not remain paraphrases for other word-choices in the corresponding positions.

It is for this reason that these relations were called transformations in linguistics, and the elementary sentences (each produced by an elementary operator, and containing no sentence as a proper part) were called kernel-sentences because they were the kernel of the natural mapping of the set of sentences onto the set of transformations (as a quotient set of it relative to having the same last operator or the same last reduction).¹⁷ Having the same last operator or reduction is an equivalence relation in the set of sentences, as is also having the same ordered entries.

In particular, the reductions give rise to a partition of the set of sentences into equivalence-classes, in each of which all sentences have the same ordered entries and objective meanings. Since almost all the reductions are optional, each equivalence class (with certain adjustments) contains one reduction-less sentence; any obligatory morphophonemics (fn. 15) is included in the reduction-less sentence. These sentences have a distinguished syntactic form (consisting of word-entry only), and the set of them is closed under the word-entry operation: any word-sequence satisfying it is such a sentence. Hence we may call this set a sub-language. Since the reductions do not materially alter the information in a sentence, this sub-language expresses all the objective information of the language.

It remains to consider the structural effects of the reductions. The connection of the reductions to low information brings into the language restrictions

¹⁷ To establish this relation, we have to take the set of sentences, $\{S\}$, as a monoid (with null sentence as identity) with *and* as binary composition in it. For any two sentences A, B , we have A and B as a new sentence C . The types of sentence-pairs C, D on which *and* is grammatically unable to operate (e.g. *Are you going? and I'm late*), are sufficiently few so that we can reasonably put their resultant as the null sentence: C and $D = \text{null}$. It follows that there is a binary composition in the set of equivalence classes $\{E\}$, with E_A and $E_B = E_{A \text{ and } B}$ (where E_X is the equivalence class to which X belongs). The natural mapping is then a homomorphism of $\{S\}$ onto $\{E\}$.

and subclasses such as had not arisen in the word-entry process. As an example, consider the reductions that produce tense (sects. 8 and IV).

Once the tenses are understood as reduced from time-order words, we can see how the operators which receive the tense affix become specialized into verbs and adjectives. Operators which had a high likelihood of occurring under *before*, *after* receive the affix directly on them, and become verbs: *He will leave after eating*, *He phoned before arriving*. Operators referring to more stable events are less likely to be time-ordered to other operators in the sentence (except to *I say*); these receive the affix indirectly, on a carrier *be*, and become adjectives: *The river is long*, *He was peculiar*. (The *was* here is from *before my saying this*.) While adjectives such as *peculiar* can occur with a *before* which relates them to some other operator in the sentence, as in *He was peculiar before she met him*, they are much less likely to do so than are the operators which become verbs, such as *phoned*.

We can now see why assigning reduced forms to high likelihood, low-information, operators creates restrictions and subclasses. The reason is that operators are complexly graded as to likelihood, whereas a reduction permits in general only two grades: receiving the reduction, and not. Hence a conventional cut-off point must be imposed on the graded operators, as a boundary for recipients of the reduction. As a result, restrictions are created: the reduction is applied restrictedly only to one member of the operator or argument class, or to a subdomain of it whose members have to be listed or characterized. Thus, subclasses are formed: a given set of operators is divided into those that receive the reduction and those that do not. For example, the directness or non-directness of tense attachment was seen to depend upon the operator's likelihood of occurring under *before*, *after*; and it serves to separate out verbs from all other operators. But while the likelihood of an operator's being under time-ordering is graded and in part uncertain, the recipients of direct attachment are a single subset. The inevitable borderline cases are decided somewhat arbitrarily but definitely; *sleep* is a verb, *ill* is an adjective. Other languages have much the same verb-adjective system, but the borderline decisions

may be different: *ill* a verb, *sleep* an adjective.¹⁸

We have here the characteristic properties of social conventions: a use-oriented graded human activity; and an organizing of some feature of it, which in so doing makes arbitrary decisions that have to be maintained by convention. More than this, many of the language conventions, even though no social interests or class control lie behind them, are institutionalized into rules, such that a departure from them is an error. It is here that rules come into grammar: to say *He illed* is a "mistake" in English, not just a nonsensical yet possible sentence such as *Vacuum ate cassettes*.

IV. RELATION TO ISSUES IN PHILOSOPHY

12. *Symbols and Terms*

The methodological approach which led to the theory of language here consisted in comparing the occurrences of segments of speech and writing—the tokens—relative to each other; it is thus close in spirit to the inscriptional approach. When the occurrence of words in the source, i.e. unreduced, discourses is found to be determined by an entry order satisfying argument-demand, we obtain a classification of words by their argument demand. This is a special case of the idea of functors in categorical grammar. However, it is this particular case that is adequate for language; not every way of defining functors would be equally suitable.

In the various operator classes defined by their argument-demands, particular kinds of terms of philosophical relevance are produced by particular low-information reductions. Thus, most abstract terms in language are nominalizations of operators (i.e. operators with argumenthood-indicators on them) whose own arguments are indefinite nouns which have been zeroed: e.g. *humility* from *the humility of people* from *the property of people's being humble*.

Disposition terms are, in language, operators under certain further aspectual (modal) operators. Suffixes, including *-ible* and its synonyms, are reduced from operators on the word to which the suffix is affixed: *X is soluble* from *X's dissolving is possible*; and suffixless disposition terms are found to result from a zero reduction of *possible* or the

¹⁸ In some cases, the borderline for a reduction is not so fixed conventionally. This becomes the ground for productivity and for marginal sentences (sentences whose acceptability is uncertain), as in such forms as *The baby took a crawl over to me*, extended from *He took a walk*, etc. Of course, all this information about which words are verbs and which are adjectives, or which verbs can be objects of *took*, would have to be given in any grammar. In the present system, these facts are relevantly organized, and some can be deduced from other facts.

like.¹⁹ In addition, since all non-indicative forms are derived by the theory above from indicative ones, it follows that the subjunctive conditional, which is semantically related to disposition terms, can be derived from conjoined indicative assertions. Furthermore, as among various occurrences of conjoined assertions, it has been found that for a conjoined assertion to have good likelihood of occurrence, or for it to express causality and necessity rather than accidentality of their connection, it is preferred that the same words repeat in the two conjoined statements, or in a chain of intervening conjoined statements which expressed well-known facts and which had therefore been zeroed as contributing little information.²⁰ Taken together, all these grammatical results suggest a syntactic characterization both of disposition terms and of the difference between laws and accidental correlations (as in counterfactuals). The fact that among disposition terms and among subjunctive conditionals there is a gradation²¹ is not surprising, once we see that their linguistic characterization rests on the degree of word-repetition in the chain of intervening sentences connecting the two statements.

A subset of words can be characterized not only by the kind of operators involved, but also by the kind of reduction which the words receive, occasioned by the particular kind of informationlessness the words have. Thus, the unique way in which *I ask you*, *I say to you*, etc., carry no information in *I ask you: Did he leave?*, etc., permits their zeroing to *Did he leave?*, etc., and thus isolates on syntactic grounds certain types of performatives.

13. Structure and Translation

The investigation of entry order as specifying, before reduction, the structure of discourses and sentences shows that discourses and sentences are not just concatenations of words constrained by one or another set of rules, but a particular sequence of operators on arguments. The meaning of a sentence is closely related to its ordered entries; therefore, translation, even under the conditions discussed by Quine in *Word and Object* and elsewhere, is not entirely indeterminate. For the most part, operators of a given argument structure in one language translate into operators of the same

argument structure in another (verbs on two nouns into verbs on two nouns, like *drink* and *boire*; verbs on a noun and a verb into verbs on a noun and a verb, like *believe* and *glauben*, etc.): there are a few important exceptions, which may be transformational, such as the possibility of translating (even in the same language) O_{nnn} (e.g. *put* in *He put the book on the table*) by O_{oo} on O_{nn} , O_{nn} (e.g. *so that on place, be*, in *He placed the book so that it was on the table*). It is true that the selectional (hence, meaning) range of each word within a class may differ widely in the two languages; and idioms and allusions differ, and there may even be circumlocutions which express a meaning of a particular sentence by a whole sequence of operators of different argument-demands from those in the given sentence. But the way in which the possible meanings of a sentence are constrained by its operator structure limits the indeterminacy of translation.

14. Information vs Truth

The predicate-operator structure of language, and such specific facts as that all non-indicative sentences (e.g. the question) are reduced from indicatives, shows that language is a structure for indicating (indeed, for transmitting) information; it does not have any basic equipment for expressing attitude and emotion, or for distinguishing truth.²² Also, when a further operator enters on a sentence, the resultant retains the meaning-contribution of the original sentence; but there is no necessary relation between the truth value of the original sentence and that of the resultant: *Zurich has a subway*, *He thinks Zurich has a subway*, *He denies that Zurich has a subway*, *Zurich may have a subway*. Not a few of the difficulties in the philosophy of language and in neighboring areas of philosophy arise from starting with the equipment which had been developed for truth systems, and using it to analyze the information system that language presents. Furthermore, where the set-theoretic equipment of logic cannot reach, the custom has been to use subtle but uncontrolled and unsystematic appeals to meaning. More adequate methods are now available.

For example the two alternative grounds for identifying meaning have always been the intension and the extension of words. However, the inscrip-

¹⁹ Note the relation of dispositional predicates to possible occurrences (grammatically: to conjoined statements about these possible occurrences) in Nelson Goodman, *Fact, Fiction, and Forecast* (Cambridge, 1955), p. 45.

²⁰ See fn. 6.

²¹ W. V. O. Quine, *Word and Object* (New York, 1960), p. 225.

²² *Op. cit.* in fn. 6 (ch. 2).

tional analysis of language reveals another basis (or correlate) for the meaning of a word: its particular selection as to which other words have good likelihood of occurring in operator or argument relation to it. This yields useful characterizations for synonymy and ambiguity, and provides criteria for metaphor and kindred semantic problems. Thus, whereas an extensional view of meaning leaves us with the problem of terms whose extension is null, the problem does not arise in language as analyzed here: *unicorn* and *centaur* have different selections (as, in a novel, do the names of the various characters), no less than *horse* and *dog*.²³ Sentences of good likelihood of occurrence would be, for example, *The unicorn is a mythical animal with one horn*, (1) *The unicorn is graceful*, *The unicorn is depicted in the Bayeux tapestries*, but not **The unicorn is half horse, half man*. Also we would find readily *The centaur is half horse, half man*, (2) *The centaur is a powerful fighter*, but not **The centaur is a mythical animal with one horn*. The asterisked sentences above can be said grammatically; but their likelihood of occurrence is very low, not because they are less true than the others (the truth of (1), (2), for example, is a problem at best), but because that is not what is likely to be said about unicorns and centaurs. One might argue that "likely to be said" is a weak basis for philosophical discrimination. But, given the datum of estimated likelihood of occurrence, the inequalities in it are stable and important entities of language structure. As words, *unicorn* and *centaur* have different selections. And while unicorns do not exist as objects, *unicorn* exists as a word. For words which have non-null extension, e.g. *horse*, the selection of the word is closely related to the properties of the referred object, horse. Similarly, the unique selection of the word *unicorn* can be semantically interpreted, without determining the truth of the properties of the object, unicorn.

Appeal to selection is not merely a device for the resolution of a particular problem. And it is not surprising that the semantic correlate in language should differ from that in set-theoretic systems where the empty set is unique in respect to what can be predicated of it. Speaking about unicorns can have meaning, and differently than speaking about horses—or about centaurs.

15. *Quantifiers, Variables, Reference*

The most glaring differences between the sen-

tential forms of logic and those of language lie in the use of variables, and of quantifiers on them. Quantifiers are not really a machinery *sui generis*. They merely use cross-reference among variables in a way that gives over-riding status to a particular predicate—namely whether the set is empty or not, or is completely covered by cross-referenced predicates; this meets the needs of material implication. In so doing, quantifiers fill a syntactic role comparable in importance to that filled in many languages by tense, which gives over-riding status to another particular predicate—namely, the time-order between conjoined predicates. The meaning that quantifiers contribute is in some cases simply not expressed in language (e.g. there is no difference between *Opossums have pouches* and *Unicorns have horns*); in other cases it is expressed by aspectual (modal) operators on the predicate (as in *Cats sometimes have small tails*, equivalent in meaning to *Some cats have small tails*); and for the rest it is expressed by modifiers on the argument (as in *some cats*); this last uses the same grammatical technique, of cross-reference among arguments, as is used in quantifiers.

More generally, cross reference among the arguments of predicates is achieved in logic by the use of variables within an explicitly or implicitly stated scope; in language, by the use of pronouns. The syntactic basis and resultant properties of these pronouns will be seen in sect. 17, from which it will be seen that the techniques of logic and those of language are not essentially different in this respect.

Reference other than cross-reference is not syntactic but semantic, and is not relevant to the present discussion.

16. *Metalinguage and Indirect Discourse*

The other great difference between language and logic, aside from that relating to information as against truth, is the fact that the metalanguage is a proper part of the language, more precisely that metalinguistic sentences are themselves sentences of the language. Clearly, *Sentences are sequences of words* is a sentence of English. Furthermore, metalinguistic operators can operate on the language-material to which they refer: "*Water*" is an English word (which linguistically is only *Water is an English word*, such quotes not being phonemic). Because the operator-selection of the metalinguistic operator is very different from that of its argu-

²³ Cf. Nelson Goodman, "On Likeness of Meaning," *Analysis*, vol. 10 (1949), pp. 1-7; Israel Scheffler, "Ambiguity, An Inscriptional Approach" in *Logic and Art*, ed. by Richard Rudner and I. Scheffler (Indianapolis, 1972), pp. 251-272.

ment, the presence of the metalinguistic operator is usually obvious from the further operators, e.g.: *Water has many impurities* as against *The word water has five letters*. Hence the metalinguistic operator is zeroable, yielding (phonemically) *Water has five letters*. The syntactic confusion of use and mention in language is thus due to the zeroability of the metalinguistic operator.

Operators of a metalinguistic character are also zeroable in other conditions, where they lead to the well-known problems of indirect discourse. First, *I ask you whether Mary will leave* is reducible to *I ask you: Will Mary leave?*, where *I ask you* is zeroable (sect. 8 and 12) to *Will Mary leave?* Similarly *I say to you that Mary will leave* is reducible to *I say to you: Mary will leave*, and then to *Mary will leave*. Other segments, such as *I said to you*, *He said to her*, are not zeroable, but the reduction from *that* to quote-intonation (as separate sentence) is available: (1) *He said that Cicero denounced Catiline* → *He said: Cicero denounced Catiline* (to use Quine's example). Now, *I say* is zeroable also in certain positions which happen to be permuted into the operand of some *X says*. Thus if we have (2) a sentence produced by the *wh-* (relative clause) operator on the sentence pair *I say that he said that someone denounced Catiline* and *I say that someone is Cicero*, then both occurrences of *I say* are zeroable, and we obtain (3) *He said that someone who is Cicero denounced Catiline*, where *someone who is* is zeroable,²⁴ yielding (4) *He said that Cicero denounced Cataline*. Had the second *I say* not been zeroed, the sentence (2) would be *He said that someone who I say is Cicero denounced Catiline* instead of becoming (3). Of course, if the speaker himself had used Cicero's name, the sentence would have been (1). If the speaker did not know that the person he named was Cicero, (1) is ambiguous with (4): the confusion is in whether he said *Cicero* or *I say Cicero*, and the confusion arises from the zeroability of *I say*.

These are straightforward and common transformations in English. What has been done here was to separate the indirect discourse problem into two parts: one, the intonational change between a direct quotation and the identically worded indirect discourse—certainly always possible; two, the changes in the indirect discourse which are due to a *which I say is X* with zeroing of *I say* (and of *which is*). This analysis can be extended to any paraphrase, introduced by *I say*, of the original speaker's words. And it can be extended to cer-

tain verbs other than *He said* (e.g. *He believes*, even *He saw*). Under other verbs there is no confusion: in (5) *I say that he tore the page* there is no different meaning as against *I say that he tore what I say is the page* which would reduce to the same word-sequence as (5).

17. Metatextual Operators and Cross-Reference

As has been noted above, sentences can carry operators which refer to locations within the sentence. This is so because every sentence has a linear order of words and, more important, a partial order of entry, and every sentence is complete before the next higher operator acts on it: e.g. *A man entered* and *A man then phoned* are complete before and operates on them to form *A man entered and a man then phoned*. An operator can therefore give information about ordered entries in its completed operand. By far the most important case of this arises when the operator gives information about sameness of word or sameness of referent at two entries in its operands: e.g. when *1.1 has same referent as 2.1* operates on *A man entered and a man then phoned*, to produce *A man entered and a man then phoned; 1.1 has the same referent as 2.1*. Here, permutations and zeroings yield *A man entered and the same (as 1.1) man then phoned*, which reduces (preserving meaning, of course) by pronouncing to *A man entered and he then phoned*, and by zeroing to *A man entered and then phoned*. Language thus has a metatextual machinery for cross-reference, for stating that a word in one argument (of an operator) is the same, or has the same referent, as a word in another argument (of that operator). This method rests essentially upon containing the material in question within the arguments of the operator—what one may call citing the material. Otherwise, the metatextual operator would not be able to name the locations of the material which is same. These locations are named as addresses relative to the metatextual operator; no other way is available for identifying locations. This citing is comparable to the scope that has to be stated for the cross-reference between variables in logical formulas, and is the basis of the comparable restrictions on cross-reference (including that inherent in quantifiers) in logic and in language—e.g. that it can be carried out only between arguments, or operators that have become arguments under other operators.

As an example of particular interest, we consider here the characterization of impredicative sen-

²⁴ *Op. cit.* in fn. 8.

tences: Aside from the light which the cross-reference system of language throws upon the syntactic machinery of cross-reference in logic, the particular conditions which make cross-reference (pronouncing) possible in language impose certain limitations upon what can be pronounced. Since nothing can be pronounced which has not been cited, as a complete linguistic entity, under the given metatextual operator, it follows that the self-referring pronoun of impredicative sentences does not exist in natural language. For example, in *This sentence is false*, the *this* cannot refer in English to the sentence of which it is part. The would-be impredicative sentence above is syntactically analyzed as *wh-* operating on *A sentence is false*; *A sentence is this (one)*; this reduces to *A sentence which is this is false*, and then to *This sentence is false*. If we now seek the antecedent of the *this*, i.e. what the *this* refers to, we find that the occurrence of *this* here has to be a reduced form of a repetition of some cited sentence. We would have (disregarding certain details) as source: S_1 . *A sentence which is S_1 is false*. This is pronounced to: S_1 . *A sentence which is this (one) is false*; then permuted to: S_1 . *This sentence is false*.

One could try to avoid this anaphoric analysis by saying that *this* is deictic in *This sentence is false*. But in the theory presented above, deictic pronouns are derived from cross-reference pronouns, in the special case when one of the cross-referenced locations is in a certain kind of zeroable metalinguistic sentence. Consider *This weather is just right*. We begin with *wh-* on *I assert that the weather is just right* and some metalinguistic sentence such as *The weather is the topic of my assertion*, producing *I assert that the weather, which is the topic of my assertion, is just right*. On this we have a metatextual operator, roughly 2.2 has the same referent as 1.1. This produces roughly

**I assert that the weather, which is the topic of my assertion here, is just right*, where *the topic of my assertion here* has a reduced form *this*. Hence **I assert that the weather which is this is just right* \rightarrow *This weather is just right*. Such derivations may seem unreal for language, but they use no more than the otherwise established equipment of grammar; and they show that even such special entities as the deictic pronouns can be derived from the simple operator system proposed here.

When the derivation above is applied to semicolon plus *wh-* on *I assert that a sentence is false* and *A sentence is the topic of my assertion*, we obtain *I assert that a sentence which is the topic of my assertion here is false*, reducible to *This sentence is false*. Here, if the sentence in question is false then my assertion is true, and if the sentence is true my assertion is false; no antinomy arises in this derivation in natural language.

The comments above relate only to the cross-reference syntactic derivation of anaphoric and deictic *this*, and to the absence of self-referring *this* in language. However, extensions of this argument may apply to the antinomy of the Liar in general. In the *Wahrheitsbegriff*, Tarski refers to Lukasiewicz's formulation of that antinomy in terms of an empirical statement (page and line on which the sentence is printed).²⁵ Henry Hiž has argued that this empirical statement is based on a deictic pronoun, even if the latter is not explicitly evident. The extension of the argument above to other forms than the explicit impredicative requires further details on cross-referencing in language.

In the same vein as above, the view of language presented here suggests other syntactic investigations that may be of interest to language-related issues in philosophy.

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²⁵ Alfred Tarski, *Logic, Semantics, Metamathematics*, tr. by J. H. Woodger, *The Concept of Truth in Formalized Languages* (Oxford, 1956), p. 158.